THE

SCHOLAR's GUIDE

TO

ARITHMETIC;

OR

A COMPLETE EXERCISE-BOOK

FOR THE

USE OF SCHOOLS.

WITH NOTES,

CONTAINING

The REASON of every RULE, demonſtrated from the moſt ſimple and evident PRINCIPLES;

TOGETHER WITH

General THEOREMS for the more extenſive USE of the SCIENCE.

K

By JOHN BONNYCASTLE,
Of the Royal Military Academy, Woolwich.

The FOURTH EDITION, corrected and improved.

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MDCCLXXXV.

P R E F A C E.

I DO not presume to offer the following Treatise of Arithmetic to the Public as a complete and finished Piece on the Subject; for to treat of the Theory and Practice fully, in a regular scientific manner, would require a much larger volume.

My principal design was to compose a short methodical tract for the purpose of teaching, and to draw up the whole in such a manner as seemed to be best adapted to the capacity and convenience of the Learner.

In pursuance of this plan, I have every where endeavoured to make the Definitions and Rules as concise and simple as possible, and to exemplify them with such Questions, in general, as are most likely to occur in Trade and Business.

The first Question of every Rule is wrought out at length, in order to shew the manner of working, and all remarks and observations are confined to the Notes; so that nothing is to be found in the Text but what it is necessary to transcribe and fix in the memory.

This last particular seems to have been greatly neglected by most of our Arithmetical Writers, and yet I am thoroughly persuaded that a proper attention to it would be of great service both to the Tutor and Scholar.

When a number of things are mixed together, which have little or no connection, they naturally create confusion, and the Learner is at a loss to discover which are to be copied and which not. This I have often found to be the case, and therefore have carefully avoided it.

When I first began this Work, I intended to have shewn the reason of every Rule from pure Arithmetical Principles; but I afterwards found, in many cases, that it would be very tedious and inconvenient. I was obliged therefore, in those instances, to have recourse to Algebra, as a more natural and elegant method of demonstration; the universal characters made use of in that science being prior to those of our present numeral notation, and by them the different Rules of Arithmetic were at first investigated.

This method may also be attended with an advantage which did not occur to me at that time; for it must naturally lead the Learner to perceive the intimate connection that subsists between Algebra and Arithmetic, and, if he is of an ingenious turn of mind, will be the most likely means of inducing him to acquire a knowledge of that Science.

It is not supposed that Learners in general can be made to attend to the reason and demonstration of every thing they perform, as that would be often tedious and impracticable. But those who intend to make themselves masters of the subject, and cannot be satisfied with knowing the rules only by *rote*, will do well to apply to the Notes,

Notes, and endeavour to become acquainted with the grounds and *rationale* of every operation.

I have been careful to give moſt of the Rules which are ſuppoſed to belong to Arithmetic, becauſe there are none of them but what are uſeful upon ſome occaſions, and any of them may be eaſily omitted when they are found unneceſſary.

The Notes likewiſe contain moſt of the uſeful Theorems that belong to this Science, which were given as a ſtill further help to the inquiſitive Pupil, and in order to make this Work a uſeful compendium to thoſe who are already acquainted with the Subject.

With reſpect to the order in which the different Rules ſhould be taught, it is a matter entirely arbitrary, and therefore no directions could be given for it; however, they are ſo diſpoſed as to have but little dependence on each other, and conſequently every Teacher is left to his own choice in that reſpect.

The Plan I have here followed ſeems to me to be the only proper one upon which a School book of this kind can be written; and I have endeavoured to render the execution of it as complete and perfect as poſſible. The manuſcript was at firſt deſigned as a Note book for my own private Scholars, and I was afterwards induced to make it public, by the hope of its being found uſeful to Tutors and Learners in general.

Dean-ſtreet, Fetter-lane,
November 3, 1781.

EXPLANATION of the CHARACTERS.

$+$ signifies plus, or addition.

$-$ minus, or subtraction.

$\times$ multiplication.

$\div$ division.

$: \; :: \; :$ proportion.

$=$ equality.

$\therefore$ therefore.

$\sqrt{}$ square root.

$\sqrt[3]{}$ cube root.

$\overline{}|^{\frac{n}{m}}$ any root, or power in general.

Division is sometimes expressed by placing the numbers one above another, in the form of a fraction; where the upper number signifies the dividend, and the lower one the divisor.

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ADVER-

ADVERTISEMENT

TO THE

THIRD EDITION.

THE motives which first lead me to the publication of this Treatise, were such as arose from a desire to promote useful Knowledge. From considerable experience, as a Teacher, I had been long persuaded, that the School-books, in general use on this Subject, were capable of many improvements. A set of concise and well-digested rules, with proper examples, appeared to be much wanted. These I endeavoured to furnish; and the kind reception this performance has met with, has convinced me, that my opinion was not ill founded, or my labour entirely useless.

In order, therefore, to accommodate the Work as much as possihle to the purpose intended, I have examined this edition with the greatest care and circumspection; and made such alterations and amendments as time and observation have furnished me with.

To every rule throughout the Book, I have given an example at full length; and added sucb remarks and explanations as cannot fail of making it easy and intelligible to the meanest capacity.

The errors, likewise, which had crept into the former editions, have been carefully attended to; and it is hoped there will be found none at present, of any material consequence. Correctness, perspicuity and brevity, are the principal objects I have had in view, through the whole performance; and if I have failed in attempting to unite them, the difficulty of the undertaking will plead my excuse, and entitle me to the candour and indulgence of the Public.

THE

THE FOLLOWING

EXTRACTS,

RELATING TO THIS

WORK,

ARE

Taken from the REVIEWS of July and September, for the Year 1780.

THE Author informs us in his Preface, that we are not to look upon this as a complete Treatise of Arithmetic; but only as a short methodical Tract, drawn up for the purpose of teaching. We assure our readers, that this is a modest account, and that many Masters may profit by what is here offered to them for the use of their Scholars.

In pursuance of this Plan, of writing a book for the use of Schools, he has been very careful to make all his definitions and rules as concise as possible, consistent with that simplicity and clearness which is absolutely necessary in things of this nature; and afterwards to exemplify those rules with a sufficient number of examples; in selecting of which, he has made choice of such as are most likely to occur in business; and has also shewn, with great clearness and perspicuity, the reason of each rule in notes, and, in some instances, has illustrated and explained the examples, when he had reason to apprehend any difficulties would be found; or where any disputes have arisen between former Authors; and, in this part of his Work, Mr. Bonnycastle has shewn great ingenuity and judgment.

By confining every thing of this nature to the notes, Mr. B. has been enabled to keep his text free from long ex-

explanations, ſo that nothing is to be found there, but what the Learner ought to tranſcribe, and fix in his memory; a matter which ſeems to have been too much neglected by moſt of thoſe Authors who have undertaken to write on the Subject of Arithmetic for the uſe of Schools.

On the whole, we ſhall not heſitate to declare that we think this little book will be found very uſeful both to the Teacher and Learner.

Monthly Review.

THE Title of this little book we have given at full length, becauſe it anſwers to its title, and does not, like many publications of this kind, profeſs more than it performs. The Author has availed himſelf of *Malcolm's Arithmetic*, and has here given us a ſcientific, as well as practical treatiſe of this uſeful branch of Learning; ſo that ſtudents of every claſs may have their deſires thoroughly gratified. The Book and its Author, of whom we know nothing, but from this performance, we recommend to the protection of the Public.

London Review.

ERRATA.

Page	*Quest.*	
37	11	for *Anſ.* 37 *lb.* 9 *gr.* read 37 *lb.* 3 *dwts.* 18 *gr.*
—	18	for *Anſ.* 934085364 *min.* 8 *ſec.* read 934085364 *m.* 48 *ſ.*
47	2	for 120 *in the Queſtion*, read 200.
60	2	for *Anſ.* 215 *cwt.* 0 *qr.* 17 *lb.* and value 577 *l.* 6 *s.* $5\frac{3}{4}$ *d.* read 215 *cwt.* 0 *qr.* 15 *lb.* and value 577 *l.* 5 *s.* $6\frac{1}{4}$ *d.*
—	3	for *Anſ.* 126 *l.* 14 *s.* $0\frac{3}{4}$ *d.* read 126 *l.* 13 *s.* $9\frac{1}{4}$ *d.*
62		*Grocer's Bill*, for *Anſ.* 83 *l.* 2 *s.* $2\frac{1}{2}$ *d.* read 86 *l.* 14 *s.* $2\frac{1}{2}$ *d.*
79	5	for *Anſ.* 12 *l.* 13 *s.* 11 *d.* read 12 *l.* 13 *s.* $11\frac{1}{2}$ *d.*
—	9	for *Anſ.* 2 *s.* 1 *d* $\frac{30}{112}$. read 2 *s.* $1\frac{1}{5}$ *d.*
106	4	for *of* $1\frac{1}{4}$ *in Queſt.* read $\frac{3}{4}$ *of* $1\frac{1}{4}$.
108	8	for *Anſ.* 3 *qr.* 3 *lb.* 10 *oz.* $12\frac{4}{9}$ *dr.* r. 3 *qr.* 3 *lb.* 1 *oz.* $12\frac{4}{9}$ *dr.*
111	7	for $\frac{3}{8}$ of a mile *in Queſt.* read $\frac{7}{8}$.
115	3	for *Anſ.* $\frac{61}{143}$ read $13\frac{61}{143}$ *poles.*
157	2	for *Anſ.* 395 *l.* read 394 *l.* 18 *s.* $6\frac{1}{4}$ *d.*
191	3	for *Anſ.* 114 *l.* read 144 *l.*
223	5	for 35 *ſt.* 10 *guild.* *in Queſt.* read 10 *guild.* 35 *ſt.*

ARITHMETIC.

ARITHMETIC is the art of computing by Numbers, and has five principal or fundamental rules for its operations; viz. Notation, Addition, Subtraction, Multiplication and Division.

NOTATION*.

Notation teacheth how to express any proposed number, either by words or characters.

To

* As it is absolutely necessary to have a perfect knowledge of our excellent method of notation, in order to understand the reasoning made use of in the following notes, I shall endeavour to explain it in as clear and concise a manner as possible.

First, then, it may be observed, that the characters by which all numbers are expressed, are these ten; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; 0 is called a cypher, and the rest, or rather all of them, are called figures or digits. The names and signification of these characters, and the origin or generation of the numbers they stand for, are as follows: 0 nothing; 1 one, or a single thing called an unit; $1 + 1 = 2$ two; $2 + 1 = 3$ three; $3 + 1 = 4$ four; $4 + 1 = 5$ five; $5 + 1 = 6$ six; $6 + 1 = 7$ seven; $7 + 1 = 8$ eight; $8 + 1 = 9$ nine; and $9 + 1 =$ ten; which has no single character; and thus by the continual addition of one all numbers are generated.

2. Besides the simple value of the figures, as above noted, they have, each, a local value, according to the following law:

Viz. In a combination of figures, reckoning from right to left, the figure in the first place represents its primitive simple value; that in the second place ten times its simple value; that in the third place a hundred times its simple value; and so on; the value of the figure

in

To read NUMBERS.

To the simple value of each figure join the name of its place, beginning at the left hand and reading towards the right.

EXAMPLES.

Read the following numbers:

37, 101, 1,107, 30791, 70079, 3306677, 111000111, 1234567890, 10203040506070809090.

To

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in each succeeding place being ten times the value of it in that immediately preceding it.

3. The names of the places are denominated according to their order. The first is called the place of units; the second, tens; the third, hundreds; the fourth, thousands; the fifth, ten thousands; the sixth, hundred thousands; the seventh, millions; and so on. Thus, in the number 3456789; 9 in the first place signifies only nine; 8 in the second place signifies eight tens or eighty; 7 in the third place is seven hundred; 6 in the fourth place is six thousand; 5 in the fifth place is fifty thousand; 4 in the sixth place is four hundred thousand; and 3 in the seventh place is three millions; and the whole number is read thus, three millions, four hundred and fifty-six thousand, seven hundred and eighty nine.

4. A cypher, though it signifies nothing of itself, yet it occupies a place, and, when set on the right hand of other figures, increases their value in the same ten-fold proportion; thus, 5 signifies only five, but 50 is five tens or fifty, and 500 is five hundred &c.

5. For the more easily reading of large numbers, they are divided into periods and half-periods, each half-period consisting of three figures; the name of the first period being units; of the second, millions; of the third, billions; of the fourth, trillions, &c. Also the first part of any period is so many units of it, and the latter part so many thousands.

The following table contains a summary of the whole doctrine.

Periods.	Quadrill.	Trillions.	Billions.	Millions.	Units
Half-per.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u.
Figures.	123,456.	789,098.	765,432.	101,234.	567,89c

To write NUMBERS.

RULE.

Write down the figures in the ſame order their values are expreſſed in, beginning at the left hand, and writing towards the right; remembering to ſupply thoſe places of the natural order with cyphers, which are omitted in the queſtion.

EXAMPLES.

Write down in figures the following numbers:

Eighty-one. Two hundred and eleven. One thouſand and thirty-nine. A million and a half. A hundred and four ſcore and five thouſand. Eleven thouſand million, eleven hundred thouſand and eleven. Thirteen billions, ſix hundred thouſand million, four thouſand and one.

SIMPLE ADDITION.

Simple Addition teacheth to collect ſeveral numbers of the ſame denomination into one total.

RULE*.

1. Place the numbers under each other, ſo that units may ſtand under units, tens under tens, &c. and draw a line under them.

2. Add

* This rule, as well as the method of proof, is founded on the known axiom, "the whole is equal to the ſum of all its parts." All that requires explaining is the method of placing the numbers and carrying for the tens; both which are evident from the nature of notation; for any other diſpoſition of the numbers would entirely alter their value; and carrying one for every ten, from an inferior line to a ſuperior, is, evidently, right, ſince an unit in the latter caſe is of the ſame value as ten in the former.

Beſides the method here given, there is another very ingenious one of proving addition by caſting out the nines, thus:

RULE. 1. Add the figures in the uppermoſt row together, and find how many nines are contained in their ſum.

 2. Reject

2. Add up the figures in the row of Units, and find how many tens are contained in their sum.

3. Set down the remainder, and carry as many ones to the next row as there are tens; with which proceed as before; and so on till the whole is finished.

Method of PROOF.

1. Draw a line below the uppermost number, and uppose it cut off.

2. Add all the rest together, and set their sum under the number to be proved. 3. Add

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2. Reject the nines, and set down the remainder directly even with the figures in the row.

3. Do the same with each of the given numbers; and set all these excesses of nine together in a line, and find their sum; then if the excess of nines in this sum, found as before, is equal to the excess of nines in the total sum the question is right.

EXAMPLE.

	Excess of nines
3782	2
5766	6
8755	7
18303	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; viz. that any number divided by 9, will leave the same remainder as the sum of its figures or digits divided by 9; which may be thus demonstrated.

Demon. Let there be any number, as 3467; this separated into its several parts becomes $3000 + 400 + 60 + 7$; but $3000 = 3 \times 1000 = 3 \times 999 + 1 = 3 \times 999 + 3$. In like manner $400 = 4 \times 99 + 4$, and $60 = 6 \times 9 + 6$. Therefore $3467 = 3 \times 999 + 3 + 4 \times 99 + 4 + 6 \times 9 + 6 + 7 = 3 \times 999 + 4 \times 99 + 6 \times 9 + 3 + 4 + 6 + 7$. And $\frac{3467}{9} = \frac{3 \times 999 + 4 \times 99 + 6 \times 9}{9} + \frac{3 + 4 + 6 + 7}{9}$. But $3 \times 999 + 4 \times 99 + 6 \times 9$ is, evidently, divisible by 9; therefore 3467 divided by 9 will leave the same remainder as $3 + 4 + 6 + 7$ divided by 9; and the same will hold for any other number whatever. *Q. E. D.*

The

3. Add this laſt found number and the uppermoſt line together, and if their ſum is the ſame as that found by the firſt addition, the queſtion is right.

EXAMPLES.

(1)	(2)	(3)
23456	22345	34578
78901	67890	3750
23456	8752	87
78901	340	328
23456	350	17
78901	78	327
307071 *Sum*	99755 *Sum*	39087 *Sum*
283615	77410	4509
307071 *Proof*	99755 *Proof*	39087 *Proof*

4. Add 8635, 2194, 7421, 5063, 2196, and 1245 together. *Anſ.* 26754.

5. Add

The ſame may be demonſtrated univerſally thus:

Demon. Let N = any number whatever, *a*, *b*, *c*, &c. the digits of which it is compoſed, and *n* = as many cyphers as *a*, the higheſt digit, is places from unity. Then $N = a$ with n, 0's $+\ b$ with $\overline{n-1}$, 0's $+\ c$ with $\overline{n-2}$, 0's, &c. by the nature of notation; $= a \times \overline{n-1}$, 9's $+ a + b \times \overline{n-2}$, 9's $+ b + c \times \overline{n-3}$, 9's $+ c$, &c. $= a \times \overline{n-1}$, 9's $+ b \times \overline{n-2}$, 9's $+ c \times \overline{n-3}$, 9's, &c. $+ a + b + c$, &c. but $a \times \overline{n-1}$, 9's $+ b \times \overline{n-2}$, 9's $+ c \times \overline{n-3}$, 9's, &c. is, plainly, diviſible by 9; therefore *N* divided by 9 will leave the ſame remainder as $a + b + c$, &c. divided by 9. *Q. E. D.*

In the very ſame manner this property may be ſhewn to belong to the number three; but the preference is uſually given to the number 9, on account of its being more convenient in practice.

Now from the demonſtration here given, the reaſon of the rule itſelf is evident; for the exceſs of nines in two or more numbers being taken ſeparately, and the exceſs of nines taken alſo out of the ſum

5. Add 246034, 298765, 47321, 58653, 64218, 5376, 9821 and 340 together. *Anf.* 730528.

6. Add 562163, 21964, 56321, 18536, 4340, 279 and 83 together. *Anf.* 663686.

7. How many fhillings are there in a crown, a guinea, a moidore, and a fix and thirty? *Anf.* 89.

8. How many days are there in the twelve calendar months? *Anf.* 365.

9. How many days are there from the 19th day of April, 1774, to the 27th day of November 1775, both days exclufive. *Anf.* 586.

SIMPLE SUBTRACTION.

Simple Subtraction teacheth to take a lefs number from a greater of the fame denomination, and thereby fhews the difference or remainder.

RULE *.

1. Place the leaft number under the greateft, fo that units may ftand under units, tens under tens, &c. and draw a line under them.

2. Begin

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of the former exceffes, it is plain this laft excefs muft be equal to the excefs of nines contained in the total fum of all thefe numbers; the parts being equal to the whole.

This rule was firft given by Dr. Wallis in his Arithmetic, publifhed anno 1657, and is a very fimple eafy method; though it is liable to this inconvenience, that a wrong operation may fometimes appear to be right; for if we change the places of any two figures in the fum, it will ftill be the fame; but then a true fum will always appear to be true by this proof; and to make a falfe one appear true, there muft be at leaft two errors, and thefe oppofite to each other; and if there are more than two errors they muft balance amongft themfelves: but the chance againft this particular circumftance is fo great, that we may as fafely truft to this proof as to any other; except, indeed, when a lazy boy, who knows the method, has a mind to tranfpofe the figures in the manner above mentioned; which muft be always guarded againft.

* *Demon.* 1. When all the figures of the leaft number are lefs than their correfpondent figures in the greater, the difference of the figures in

2. Begin at the right hand, and take each figure in the lower line from the figure above it, and ſet down the remainder.

3. If the lower figure is greater than that above it, add ten to the upper number; from which number, ſo increaſed, take the lower, and ſet down the remainder, carrying one to the next lower number; with which proceed as before, and ſo on till the whole is finiſhed.

Method of PROOF.

Add the remainder to the leaſt number, and if the ſum is equal to the greateſt, the work is right.

EXAMPLES.

	(1)		(2)		(3)
From	3287625	From	5327467	From	1234567
Take	2343756	Take	1008438	Take	345678
Rem.	943869	*Rem.*	4319029	*Rem.*	888889
Proof.	3287625	*Proof.*	5327467	*Proof.*	1234567

4. From 2637804 Take 2376982. *Anſ.* 260822
5. From 3762162 Take 826541. *Anſ.* 2935621
6. From 78213606 Take 27821890. *Anſ.* 50391716

7. The

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in the ſeveral like places muſt altogether make the true difference ſought; becauſe as the ſum of the parts is equal to the whole, ſo muſt the ſum of the differences of all the ſimilar parts be equal to the difference of the whole.

2. When any figure of the greateſt number is leſs than its correſpondent figure in the leaſt, the ten which is added by the rule is the value of an unit in the next higher place, by the nature of notation; and the one that is added to the next place of the leaſt number is to diminiſh the correſpondent place of the greater accordingly; which is only taking from one place and adding as much to another, whereby the total is never changed. And by this means the greater number is reſolved into ſuch parts as are each greater than, or equal to, the ſimilar parts of the leſs; and the difference of the correſponding figures, taken together, will, evidently, make up the difference of the whole. *Q. E. D.*

The truth of the method of proof is evident: for the difference of two numbers added to the leaſt is, manifeſtly, equal to the greater.

7. The Arabian method of notation was first known in England about the year 1150, how long is it since, to this present year 1776? *Ans.* 626 *years.*

8. Sir Isaac Newton was born in the year 1642, and died in 1727, how old was he at the time of his decease. *Ans.* 85 *years.*

9. A grant of the crown, *anno domini*, 1237, was forfeited 137 years before the revolution in 1688; how long did the same subsist? *Ans.* 314 *years.*

SIMPLE MULTIPLICATION.

Simple Multiplication is a compendious method of addition, and teacheth to find the amount of any given number of one denomination, by repeating it any proposed number of times.

The number to be multiplied is called the *multiplicand.*

The number you multiply by is called the *multiplier.*

The number found from the operation is called the *product.*

Both the multiplier and multiplicand are, in general, called *terms* or *factors.*

RULE*.

1. Place the multiplier under the multiplicand, so that units may stand under units, tens under tens, &c. and draw a line under them.

2. Begin

* *Demon.* 1. When the multiplier is a single digit it is plain that we find the product; for by multiplying every figure, that is, every part of the multiplicand, we multiply the whole; and by writing down the products that are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens to the product of the next place, is only gathering together the similar parts of the respective products, and is, therefore, the same thing, in effect, as though we wrote down the multiplicand as often as the multiplier expresses, and added them together: for the sum of every column is the product of the figures in the place of that column; and

2. Begin at the right hand, and multiply the whole multiplicand severally by each figure in the multiplier, setting down the first figure of every line directly under the figure you are multiplying by, and carrying for the tens as in addition.

3. Add all the lines together, and their sum is the product.

Method of PROOF.

Make the former multiplicand the multiplier, and the multiplier the multiplicand, and proceed as before; and if this product is equal to the former, the question is right.

EXAM-

---

and these products collected together are, evidently, equal to the whole required product.

2. If the multiplier is a number made up of more than one digit. After we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and find, after the same manner, the product of the multiplicand by the second figure of the multiplier; but as the figure we are multiplying by stands in the place of tens, the product must be ten times its simple value; and therefore the first figure of this product must be placed in the place of tens; or, which is the same thing, directly under the figure we are multiplying by. And proceeding in this manner separately with all the figures of the multiplier, it is evident that we shall multiply all the parts of the multiplicand by all the parts of the multiplier; or, the whole of the multiplicand by the whole of the multiplier; therefore these several products being added together will be equal to the whole required product. *Q. E. D.*

The reason of the method of proof depends upon this proposition, "that if two numbers are to be multiplied together, either of them may be made the multiplier, or the multiplicand, and the product will be the same". A small attention to the nature of numbers will make this truth evident: for $3 \times 7 = 21 = 7 \times 3$; and in general $3 \times 4 \times 5 \times 6$, &c. $= 4 \times 3 \times 6 \times 5$, &c. without any regard to the order of the terms: and this is true of any number of factors whatever.

The following examples are subjoined to make the reason of the rule appear as plain as possible.

137565

EXAMPLES.

(1)		(2)	
Mult.	23456787454	Mult.	32745654473
	by 7		by 234
	164197512178 Product.		130982617892
			98236963419
			65491308946
		Product.	7662483146682

3. Multiply 32745675474 by 2. *Ans.* 65491350948
4. Nultiply 374328756432 by 3. *Ans.* 1122986269296
5. Multiply 5806342748 by 4. *Ans.* 23225370992
6. Multiply 84356745674 by 5. *Ans.* 421783728370

---

(1)			(2)		
37565			1375435		
5			4567		
25	=	5 × 5	9628045	=	7 times the mult.
30	=	60 × 5	8252610	=	60 times ditto.
25	=	500 × 5	6877175	=	500 times ditto.
35	=	7000 × 5	5501740	=	4000 times ditto.
15	=	30000 × 5			
187825	=	37565 × 5	6281611645	=	4567 times ditto.

Besides the preceding method of proof, there is another very convenient and easy one by the help of that peculiar property of the number 9, mentioned in addition; which is performed thus:

RULE. 1. Cast the nines out of the two factors, as in addition, and set down the remainders.

2. Multiply the two remainders together, and if the excess of nines in their product is equal to the excess of nines in the total product, the question is right.

EXAMPLE.

4215—3 = excess of 9's in the multiplicand.
878—5 = ditto in the multiplier.

33720
29505
33720

3700770—6 = ditto in the product, = excess of 9's in 3 × 5.

Demon.

7. Multiply 274567846473 by 6. *Ans.* 1647407078838
8. Mult. 54328432847 by 8. *Ans.* 434627462776
9. Mult. 8643597 by 9. *Ans.* 77792373
10. Mult. 796534289 by 11. *Ans.* 8761877179
11. Mult. 3274656461 by 12. *Ans.* 39295877532
12. Mult. 7324687567 by 15. *Ans.* 109870313505
13. Mult. 94713761 by 18. *Ans.* 1704847698
14. Mult. 273580961 by 23. *Ans.* 6292362103
15. Mult. 27501976 by 271. *Ans.* 7453035496
16. Mult. 82164973 by 3027. *Ans.* 248713373271
17. Mult. 6247386495 by 27356. *Ans.* 170903504957220
18. Mult. 8496427 by 874359. *Ans.* 7428927415293
19. Mult. 467853798 by 6839754. *Ans.* 3200004886285692
20. Mult. 123456789 by 123456789. *Ans.* 15241578750190521.

CONTRACTIONS.

I. *When there are cyphers to the right hand of one or both the numbers to be multiplied.*

RULE.

Proceed as before, neglecting the cyphers, and to the right hand of the product place as many cyphers as are in both the numbers.

EXAM-

Demon of the Rule. Let M and N be the number of 9's in the factors to be multiplied, and a and b what remains; then $\overline{M+a}$ and $\overline{N+b}$ will be the numbers themselves, and their product is $M \times N + \overline{M \times b} + \overline{N \times a} + \overline{a \times b}$; but the three first of these products are each a precise number of 9's, because one of their factors is so: therefore these being cast away there remains only $a \times b$; and if the 9's are also cast out of this, the excess is the excess of 9's in the total product; but a and b are the excesses in the factors themselves, and $a \times b$ their product; therefore the rule is true. *Q. E. D.*

This method is liable to the same inconvenience with that in addition.

Multiplication may also, very naturally, be proved by division; for the product being divided by either of the factors will, evidently, give the other; but it would have been contrary to good method to have given this rule in the text, because the pupil is supposed, as yet, to be unacquainted with division.

EXAMPLES.

1. Multiply 1234500 by 7500.

12345
75
61725
86415
9258750000 *the Product.*

2. Multiply 461200 by 72000. *Ans.* 33206400000
3. Multiply 815036000 by 70300 *Ans.* 57297030800000

II. *When the multiplier is the product of two or more numbers in the table.*

RULE*.

Multiply continually by those parts instead of the whole number at once.

EXAMPLES.

1. Multiply 123456789 by 25.

123456789
5
617283945
5
3086419725 *the Product.*

2. Multiply 364111 by 56. *Ans.* 20390216
3. Multiply 46123101 by 72. *Ans.* 3320863272
4. Multiply 7128368 by 96. *Ans.* 684323328
5. Multiply 61835720 by 132. *Ans.* 8162315040
6. Multiply 123456789 by 1440. *Ans.* 177777776160

* The reason of this method is obvious; for any number multiplied by the component parts of another number must give the same product, as though it were multiplied by that number at once: thus, in example the second, 7 times the product of 8, multiplied into the given number, makes 56 times that given number, as plainly as 7 times 8 makes 56.

SIMPLE

SIMPLE DIVISION.

Simple Division teacheth to find how often one number is contained in another of the same denomination, and thereby performs the work of many subtractions.

The number to be divided is called the *dividend.*

The number you divide by is called the *divisor.*

The number of times the dividend contains the divisor is called the *quotient.*

If the dividend contains the divisor any number of times, and some part or parts over, those parts are called the *remainder.*

RULE*.

1. On the right and left of the dividend draw a curved line, and write the divisor on the left hand, and the quotient as it arises on the right.

2. Find

---

* According to the rule, we resolve the dividend into parts, and find by trial the number of times the divisor is contained in each of those parts; the only thing then which remains to be proved is, that the several figures of the quotient, taken as one number, according to the order in which they are placed, is the true quotient of the whole dividend by the divisor; which may be thus demonstrated:

Demon. The complete value of the first part of the dividend, is, by the nature of notation, 10, 100, or 1000, &c. times the value of which it is taken in the operation; according as there are 1, 2, or 3, &c. figures standing before it; and consequently the true value of the quotient figure belonging to that part of the dividend is also 10, 100, or 1000, &c. times its simple value. But the true value of the quotient figure belonging to that part of the dividend, found by the rule, is also 10, 100, or 1000, &c. times its simple value: for there are as many figures set before it as the number of remaining figures in the dividend. Therefore this first quotient figure taken in its complete value, from the place it stands in, is the true quotient of the divisor in the complete value of the first part of the dividend. For the same reason all the rest of the figures of the quotient, taken according to their places, are each the true quotient of the divisor in the complete value of the several parts of the dividend belonging to each; because, as the first figure on the right hand of each succeeding part of the dividend has a less number of figures by one standing before it, so ought their

2. Find how many times the divisor may be had in as many figures of the dividend as are just necessary, and write the number in the quotient.

3. Multiply the divisor by the quotient figure, and set the product under that part of the dividend used.

4. Subtract the last found product from that part of the dividend under which it stands, and to the right hand of the remainder bring down the next figure of the dividend; which number divide as before; and so on, till the whole is finished.

Method

---

their quotients to have; and so they are actually ordered: consequently taking all the quotient figures in order as they are placed by the rule, they make one number, which is equal to the sum of the true quotients of all the several parts of the dividend; and is, therefore, the true quotient of the whole dividend by the divisor. *Q. E. D.*

To leave no obscurity in this demonstration I shall illustrate it by an example.

EXAMPLE.

Divisor 36)85609 dividend.

1st. part of the dividend.	85000		
36 × 2000 =	72000	— —	2000 the 1st. quotient.
1st. remainder —	13000		
add —	600		
2d. part of the dividend.	13600		
36 × 300 =	10800	— —	300 the 2d. quotient.
2d. remainder —	2800		
add —	00		
3d. part of the dividend	2800		
36 × 70 =	2520	— —	70 the 3d. quotient.
3d. remainder —	280		
add —	9		
4th. part of the dividend	289		
36 × 8 =	288	— —	8 the 4th. quotient.
Last remainder —	1	—	2378 sum of all the quotients, or the answer.

Expla.

Method of PROOF.

Multiply the quotient by the divisor, and this product added to the remainder will be equal to the dividend, when the work is right.

Ex-

Explanation. It is evident that the dividend is resolved into these parts, 85000+600+00+9: for the first part of the dividend is considered only as 85, but yet it is truly 85000; and therefore its quotient instead of 2 is 2000, and the remainder 13000; and so of the rest, as may be seen in the operation.

When there is no remainder to a division, the quotient is the absolute and perfect answer to the question; but where there is a remainder, it may be observed, that it goes so much towards another time as it approaches to the divisor: thus, if the remainder be a fourth part of the divisor, it will go one fourth of a time more; if half the divisor, it will go the half of a time more; and so on. In order, therefore, to complete the quotient, put the last remainder at the end of it, above a small line, and the divisor below it.

It is sometimes difficult to find how often the divisor may be had in the numbers of the several steps of the operation: the best way will be to find how often the first figure of the divisor may be had in the first, or two first, figures of the dividend, and the answer made less by one or two is generally the figure wanted: besides, if after subtracting the product of the divisor and quotient from the dividend, the remainder be equal to, or exceed the divisor, the quotient figure must be increased accordingly.

If, when you have brought down a figure to the remainder it is still less than the divisor, a cypher must be put in the quotient, and another figure brought down, and then proceed as before.

The reason of the method of proof is plain: for since the quotient is the number of times the dividend contains the divisor, the product of the quotient and divisor must, evidently, be equal to the dividend.

There are several other methods made use of to prove division: the best and most useful are these following.

Rule I. Subtract the remainder from the dividend, and divide this number by the quotient, and the quotient found by this division will be equal to the former divisor, when the work is right.

The reason of this rule is plain from what has been observed above.

Mr. Malcolm, in page 71 of his Arithmetic, has been drawn into a mistake concerning this method of proof, by making use of particular numbers instead of a general demonstration. He says, the dividend being divided by the integral quotient, the quotient of this division will be equal to the former divisor with the same remainder. This is true in some particular cases; but it will not hold when the

remainder

EXAMPLES.

(1)

5)135457284565

27091456913

(2)

365)123456789(338237
1095
1395
1095
3006
2920
867
730
1378
1095
2839
2555
284

3. Divide

remainder is greater than the quotient, as may be easily demonstrated; but one instance will be sufficient; thus, 17 divided by 6 gives the integral quotient 2 and remainder 5; but 17 divided by 2 gives the integral quotient 8 and remainder 1. This shews how cautious we ought to be in deducing general rules from particular examples.

Rule II. Add the remainder and all the products of the several quotient figures by the divisor together, according to the order in which they stand in the work, and the sum will be equal to the dividend, when the work is right.

The reason of this rule is extremely obvious: for the numbers that are to be added are the products of the divisor by every figure of the quotient separately, and each possesses by its place its complete value, therefore the sum of the parts, together with the remainder, must be equal to the whole.

Rule III. Subtract the remainder from the dividend, and what remains will be equal to the product of the divisor and quotient; which may be proved by casting out the nines as was done in multiplication.

This rule has been already demonstrated in multiplication.

To avoid obscurity, I shall give an example proved according to all the different methods.

Ex-

3. Divide 3756789275474 by 2. *Ans.* 1878394637737
4. Divide 5474857647651 by 3. *Ans.* 1824952549217
5. Divide 653783754732 by 4. *Ans.* 163445938688
6. Divide 2345678964 by 6. *Ans.* 390946494
7. Divide 12345678900 by 7. *Ans.* 1763668414 $\frac{2}{7}$
8. Divide 9876543210 by 8. *Ans.* 1234567901 $\frac{2}{8}$
9. Divide 1357975313 by 9. *Ans.* 150886145 $\frac{8}{9}$
10. Divide 570196382 by 12. *Ans.* 47516365 $\frac{2}{12}$
11. Divide 3217684329765 by 17. *Ans.* 189275548809 $\frac{12}{17}$
12. Divide 3211473 by 27. *Ans.* 118943 $\frac{12}{27}$
13. Divide 137896254 by 97 *Ans.* 1421610 $\frac{84}{97}$
14. Divide 1406373 by 108. *Ans.* 13021 $\frac{105}{108}$
15. Divide 3405657254 by 345. *Ans.* 9871470 $\frac{104}{345}$
16. Divide 5713070046 by 678. *Ans.* 8426357
17. Divide 293839455936 by 8405. *Ans.* 34960078 $\frac{346}{8405}$
18. Divide 4637064283 by 57606. *Ans.* 80496 $\frac{11707}{57606}$
19. Divide 352107193214 by 210472. *Ans.* 1672940 $\frac{165534}{210472}$
20. Divide 55800117260617 6724 by 2708630425.
Ans. 206008604—24 *rem.*

EXAMPLE.

```
87)123456789(1419043          123456789
   87*             87                48
   ---        -------         ---------
   364        9933301   1419043)123456741(87 Proof by Division.
   348*      11352344            11352344
   ---             48            --------
  .165        -------             9933301
  . 87*     123456789 Proof by Mult. 9933301
  .---      ---------             -------
  . 786
  . 783*
  .---
  ...378
  ...348*
  ...---
  .....309
  .....261*
  .....---
  .....  48*
  ----------
  123456789 Proof by Addition.
  ---------
```

Proof by casting out the 9's. { 4 is the excess of 9's in the quotient.
6 ditto. in the divisor.
6 ditto. in 4 × 6, which is also the excess of 9's in (123456741) the dividend made less by the remain.

For illustration, we need only refer to the example; except for the proof by addition; where it may be remarked, that the asterisms shew the numbers to be added, and the dotted lines their order.

I. To

CONTRACTIONS.

I. To divide by any number with cyphers annexed.

RULE*.

Cut off the cyphers from the divisor, and the same number of digits from the right hand of the dividend; then divide the remaining figures by each other, as usual, and the quotient is the answer; and what remains, wrote before the figures cut off, is the true remainder.

EXAMPLES.

1. Divide 310869017 by 7100.

```
71,00)3108690,17(43784 2617/7100 the quotient.
      284
      ---
       268
       213
       ---
        556
        497
        ---
         599
         568
         ---
          310
          284
          ---
          2617
```

2. Divide 7380964 by 23000. *Ans.* 320 20964/23000

3. Divide 29628754963 by 35000. *Ans.* 846535 19963/35000

II. *When*

* The reason of this contraction is easy to conceive: for the cutting off the same figures from each, is the same as dividing each of them by 10, 100, 1000, &c. and it is evident, that as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in a like part of the dividend:—This method is only to avoid a needless repetition of cyphers, which would happen in the common way, as may be seen by working an example at large.

II. *When the divisor is the product of two or more small numbers in the table.*

RULE*.

Divide continually by those numbers instead of the whole divisor at once.

Ex-

* This follows from contraction the 3d. in multiplication, of which it is only the converse; for the third part of the half of any thing is, evidently, the same as the sixth part of the whole; and so of any other number. I have omitted saying any thing, in the rule, about the method of finding the true remainders; for, as the learner is supposed, at present, to be unacquainted with the nature of fractions, it would be improper to introduce them in this part of the work, especially as the integral quotient is sufficient to answer most of the purposes of practical division. However, as the quotient is incomplete without this remainder, and in some computations it is necessary it should be known, I shall here shew the manner of finding it, without any assistance from fractions.

Rule. Multiply the quotient by the divisor, and subtract the product from the dividend, and the result will be the true remainder.

The truth of this is extremely obvious; for if the product of the divisor and quotient, added to the remainder, be equal to the dividend, their product taken from the dividend must leave the remainder.

The rule which is most commonly made use of is this:

Rule. Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on, till you have gone through all the divisors and remainders to the first.

EXAMPLE.

9)64865 divided by 144.		1 the last remainder.
4)7207—2	Mult.	4 the preceding divisor.
4)1801—3		4
450—1	Add	3 the 2d. remainder.
		7
	Mult.	9 the first divisor.
		63
	Add	2 the first remainder.
Ans. $450\frac{65}{144}$		65

To

EXAMPLES.

1. Divide 31046835 by 56.

$$\begin{array}{r} 7)\underline{31046835} \\ 8)\underline{4435262}-1 \\ 554407-6 \end{array}$$ *the quotient.*

2. Divide 7014596 by 72. *Ans.* $97424\frac{68}{72}$
3. Divide 5130652 by 132. *Ans.* $38868\frac{76}{132}$
4. Divide 83016572 by 240. *Ans.* $345902\frac{92}{240}$

III. *To perform division more concisely than by the general rule.*

RULE*.

Multiply the divisor by the quotient figures as before, and subtract each figure of the product as you produce it, always remembring to carry as many to the next figure as were borrowed before.

EXAMPLES.

1. Divide 3104675846 by 833.

$$\begin{array}{l} 833)3104675846)3727101\frac{8}{833} \\ \quad\quad 6056 \\ \quad\quad\;\; 2257 \\ \quad\quad\quad 5915 \\ \quad\quad\quad\; 848 \\ \quad\quad\quad\;\; 1546 \\ \quad\quad\quad\quad\; 713 \end{array}$$ *the quotient.*

2. Divide 29137062 by 5317. *Ans.* $5479\frac{5219}{5317}$
3. Divide 62015735 by 7803. *Ans.* $7947\frac{5294}{7803}$
4. Divide 432756284563574 by 873469.

Ans. $495445498\frac{873012}{873469}$.

To explain this rule from the example, we may observe that every unit of the 1st. quotient may be looked upon as containing 9 of the units in the given dividend; consequently every unit that remains will contain the same; therefore this remainder must be multiplied by 9 in order to find the units it contains of the given dividend. Again, every unit in the next quotient will contain 4 of the preceding ones, or 36 of the first, that is, 9 times 4; therefore what remains must be multiplied by 36; or, which is the same thing, by 9 and 4 continually. Now, this is the same as the rule; for instead of finding the remainders separately, they are reduced from the bottom upwards, step by step, to one another, and the remaining units of the same class taken in as they occur.

* The reason of this rule is the same as that of the general rule, p. 13.

COMPOUND ADDITION.

Compound Addition teacheth to collect several numbers of different denominations into one total.

RULE*.

1. Place the numbers so that those of the same denomination may stand directly under each other, and draw a line below them.

2. Add up the figures in the lowest denomination, and find how many ones of the next higher denomination are contained in their sum.

3. Write down the remainder, and carry the ones to the next denomination; with which proceed as before; and so on, through all the denominations to the highest, whose sum must be all written down; and this sum, together with the several remainders, is the total sum required.

The method of proof is the same as in simple addition.

EXAMPLES OF MONEY.

l.	s.	d.	l.	s.	d.	l.	s.	d.
17	13	4	84	17	$5\frac{1}{2}$	175	10	10
13	10	2	75	13	$4\frac{1}{4}$	107	13	$11\frac{1}{4}$
10	17	3	51	17	$8\frac{3}{4}$	89	18	10
8	8	7	20	10	$10\frac{1}{4}$	75	12	$2\frac{1}{4}$
3	3	4	17	15	$4\frac{1}{2}$	3	3	$3\frac{3}{4}$
-	8	8	10	10	11	1	-	$-\frac{1}{2}$
54	1	4	261	5	$8\frac{1}{4}$	452	19	$2\frac{1}{4}$
36	8	0	176	8	$2\frac{3}{4}$	277	8	$4\frac{1}{4}$
54	1	4	261	5	$8\frac{1}{4}$	452	19	$2\frac{1}{4}$

* The reason of this rule is evident from what has been said in simple addition: for, in addition of money, as 1 in the pence is equal to 4 in the farthings; 1 in the shillings to 12 in the pence; and 1 in the pounds to twenty in the shillings; therefore, carrying as directed, is nothing more than providing a method of digesting the money arising from each column properly in the scale of denominations; and this reasoning will hold good in the addition of compound numbers of any denomination whatsoever.

l.	s.	d.
173	13	5
87	17	$7\frac{3}{4}$
75	18	$7\frac{1}{2}$
25	17	$8\frac{1}{4}$
10	10	$10\frac{1}{2}$
2	5	$7\frac{1}{4}$

l.	s.	d.
705	17	$3\frac{1}{2}$
354	17	$2\frac{3}{4}$
175	17	$8\frac{3}{4}$
87	19	$7\frac{1}{2}$
52	12	$7\frac{3}{4}$
27	10	$5\frac{1}{4}$

l.	s.	d.
1275	12	$4\frac{1}{4}$
700	10	$10\frac{1}{2}$
25	13	$3\frac{3}{4}$
5	17	$7\frac{3}{4}$
—	18	8
—	17	—

TROY WEIGHT.

lb.	oz.	dwt.	gr.
17	3	15	11
13	2	13	13
15	3	14	14
13	10	—	—
12	1	—	17
—	—	13	14

lb.	oz.	dwt.	gr.
14	10	13	20
13	10	18	21
14	10	10	10
10	1	2	3
1	4	4	4
—	1	19	—

lb.	oz.	dwt.	gr.
27	10	17	18
17	10	13	13
13	11	13	1
10	1	—	2
4	4	3	3
2	—	—	1

APOTHECARIES WEIGHT.

lb.	oz.	dr.	sc.	gr.
3	5	7	2	17
2	7	4	2	18
1	7	5	1	10
1	7	5	2	10
2	7	3	2	17
2	6	1	1	10

lb.	oz.	dr.	sc.	gr.
4	5	6	1	13
2	7	5	2	17
1	6	1	2	7
3	4	2	1	4
2	2	1	2	—
3	1	1	1	—

lb.	oz.	dr.	sc.	gr.
5	4	3	1	10
4	3	2	2	18
3	2	1	1	17
4	2	1	11	4
3	2	-	-	10
1	-	7	2	2

cwt.	qr.	lb.	oz.	dr.
15	2	15	15	15
13	2	17	13	14
12	2	13	14	14
10	1	17	15	—
12	1	10	—	10
10	1	12	1	7

T.	cwt.	qr.	lb.	oz.	dr.
2	17	3	13	8	7
2	13	3	14	8	8
1	16	-	10	-	5
2	13	-	—	1	7
1	14	1	1	2	2
4	16	1	7	7	5

T.	cwt.	qr.	lb.	oz.	dr.
3	13	2	10	7	7
2	14	1	17	6	6
4	17	-	14	-	6
2	13	-	12	7	7
3	13	-	10	4	4
5	—	2	12	8	8

LONG MEASURE.

mls.	fu.	pls.	yd.	ft.	in.
37	3	14	2	1	5
28	4	17	3	2	10
17	4	4	3	1	2
10	5	6	3	1	7
29	2	2	2	-	3
30	-	-	4	-	2

mls.	fu.	pls.	yd.	ft.	in.
28	2	13	1	1	4
39	1	17	2	2	10
28	1	14	2	2	—
48	1	17	2	2	7
37	1	29	-	-	3
2	-	20	-	2	1

mls.	fu.	pls.	yd.	ft.	in.
28	3	7	2	-	7
30	-	-	1	-	7
27	6	30	2	2	-
7	6	20	2	1	-
5	2	-	-	2	10
-	7	10	-	2	2

CLOTH MEASURE.

yds.	qrs.	nls.	in.
120	3	1	1
38	2	-	1
28	2	-	2
38	2	-	2
28	2	3	-
118	3	2	2

En. ells.	qrs.	nls.	in.
207	2	2	1
58	2	2	-
78	-	1	1
21	3	3	2
20	-	2	2
—	3	-	2

Fl. ells.	qrs.	nls.	in.
200	2	1	1
57	1	1	2
28	1	1	1
21	-	-	2
38	-	3	1
—	-	2	2

ac.	ro.	p.
324	3	37
127	2	27
87	2	20
75	2	17
47	3	2
—	1	10

ac.	ro.	p.
370	2	26
217	2	21
87	2	20
17	3	18
9	—	—
1	—	17

ac.	ro.	p.
1775	1	29
752	2	27
75	1	27
17	2	21
2	—	17
1	1	15

WINE MEASURE.

T.	hhds.	gal.	qt.	p.
17	2	10	2	1
10	2	27	2	1
8	3	24	2	—
5	2	27	2	—
2	1	17	1	1
—	3	29	2	1

T.	hhds.	gal.	qt.	p.
27	1	3	1	1
24	—	13	—	1
21	3	37	—	—
10	2	35	1	1
8	2	25	1	1
2	2	35	2	—

T.	hhds.	gal.	qt.	p.
37	1	2	1	1
27	—	27	3	1
20	2	24	—	—
20	1	29	2	1
—	3	39	2	1
—	2	37	2	1

ALE and BEER MEASURE.

hhds.	galls.	qts.	p.
21	2	2	1
21	20	3	—
21	21	2	—
10	10	2	—
3	3	3	—
—	2	2	—

hhds.	galls.	qts.	p.
27	3	2	1
25	10	2	—
21	13	—	—
10	17	—	—
8	7	2	—
4	2	2	1

hhds.	galls.	qts.	p.
30	20	3	1
28	29	2	—
20	20	—	—
18	18	1	1
17	17	—	—
6	6	1	1

DRY

DRY MEASURE.

L.	qrs.	bu.	pe.	galls.
5	5	2	3	1
3	2	3	3	1
2	2	3	2	1
1	2	2	2	-
2	1	7	3	-
-	5	6	2	-

TIME.

Y.	mo.	we.	da.	ho.	mi.	sec.
27	9	2	6	23	25	25
20	7	2	5	20	36	30
18	7	3	4	5	6	7
14	-	1	-	21	22	23
10	-	-	2	4	5	5
8	-	2	4	-	3	38

COMPOUND SUBTRACTION.

Compound Subtraction teacheth to find the difference of any two numbers of different denominations.

RULE*.

1. Place the least number under the greatest, so that those parts which are of the same denomination may stand directly under each other, and draw a line below them.

2. Begin at the right hand, and take each figure of the lower line from the figure standing above it, and set down their remainders below them.

3. But if the figure below is greater than that above it, increase the upper number by as many as make one of the next higher denomination, and from this sum take the figure in the lower line, and set down the remainder as before.

4. Carry the unit borrowed to the next number in the lower line, and subtract as before; and so on, till the whole is finished; and all the several remainders taken

* The reason of this rule will readily appear from what was said in simple subtraction; for the borrowing depends upon the very same principle, and is only different, as the numbers to be subtracted are of different denominations.

C

together

together as one number will be the whole difference required.

The method of proof is the ſame as in ſimple ſubtraction.

EXAMPLES of MONEY.

	l.	s.	d.	l.	s.	d.	l.	s.	d.
From	275	13	4	454	14	$2\frac{3}{4}$	274	14	$2\frac{1}{4}$
Take	176	16	6	276	17	$5\frac{1}{2}$	85	15	$7\frac{3}{4}$
Rem.	98	16	10	177	16	$9\frac{1}{4}$	188	18	$6\frac{1}{2}$
Proof	275	13	4	454	14	$2\frac{3}{4}$	274	14	$2\frac{1}{4}$

TROY WEIGHT.

	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
From	7	3	14	11	27	2	10	20	29	3	14	5
Take	3	7	15	20	20	3	5	21	20	7	15	7
Rem.												
Proof												

APOTHECARIES WEIGHT.

	lb.	oz.	dr.	ſc.	gr.	lb.	oz.	dr.	ſc.	gr.	lb.	oz.	dr.	ſc.	gr.
From	11	4	7	–	14	2	3	6	1	10	5	1	3	2	19
Take	—	3	7	1	15	1	8	7	2	12	2	2	5	1	—
Rem.															
Proof															

AVOIR-

AVOIRDUPOIS WEIGHT.

	Cwt.	*qr.*	*lb.*	*oz.*	*dr.*	*Cwt.*	*qr.*	*lb.*	*oz.*	*dr.*	*Cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
From	5	-	17	5	9	22	2	13	4	8	21	1	7	6	13
Take	3	3	21	1	7	20	1	17	6	6	13	-	8	8	14
Rem.															
Proof															

LONG MEASURE.

	M.	*fur.*	*pls.*	*yds.*	*ft.*	*in.*	*M.*	*fu.*	*pls.*	*yd.*	*ft.*	*in.*	*M.*	*fu.*	*pls.*	*yd.*	*ft.*	*in.*
From	14	3	17	1	2	1	70	7	13	1	1	2	70	3	10	-	-	3
Take	10	7	30	2	-	10	20	-	14	2	2	7	17	3	11	1	1	7
Rem.																		
Proof																		

CLOTH MEASURE.

	Y.	*qr.*	*nls.*	*En. ells.*	*qr.*	*nls.*	*Fl. ells.*	*qr.*	*nls.*	*in.*
From	27	3	3	127	2	-	270	1	-	1
Take	10	2	2	78	3	3	140	2	2	2
Rem.										
Proof										

LAND MEASURE.

	A.	*ro.*	*pls.*	*A.*	*ro.*	*pls.*	*A.*	*ro.*	*pls.*
From	29	2	27	27	1	25	125	-	39
Take	21	-	28	14	-	—	87	3	1
Rem.									
Proof									

WINE MEASURE.

	T.	*hhds.*	*gall.*	*qrs.*	*pt.*	*hhds.*	*gall.*	*qrt.*	*pt.*	*hhds.*	*gall.*	*qrt.*
From	2	3	20	3	1	2	21	2	-	13	-	1
Take	1	2	17	-	-	-	—	3	1	10	27	1
Rem.												
Proof												

ALE and BEER MEASURE.

	hhds.	*fir.*	*gall.*	*qrt.*	*p.*	*hhds.*	*fir.*	*gall.*	*p.*	*hhds.*	*fir.*	*gall.*	*pt.*
From	27	2	2	2	1	29	2	3	4	27	3	2	2
Take	10	3	4	3	-	20	2	4	5	10	-	-	3
Rem.													
Proof													

DRY MEASURE.

	L.	*qr.*	*bu.*	*pe.*	*ga.*	*pt.*	*L.*	*qr.*	*bu.*	*pe.*	*gall.*	*L.*	*qr.*	*bu.*	*pe.*	*gal.*
From	9	4	7	1	1	1	13	3	5	2	1	27	1	2	-	-
Take	2	-	5	3	-	7	2	3	7	-	-	10	2	2	1	1
Rem.																
Proof																

TIME.

	mo.	*we.*	*da.*	*ho.*	*min.*	*mo.*	*we.*	*da.*	*ho.*	*min.*	*mo.*	*we.*	*da.*	*ho.*	*m.*
From	17	2	5	17	26	37	1	-	13	1	71	-	-	-	5
Take	10	-	-	18	18	15	2	-	15	14	17	-	5	5	7
Rem.															
Proof															

COM-

COMPOUND MULTIPLICATION.

Compound Multiplication teacheth to find the amount of any given number of different denominations by repeating it any propoſed number of times.

R U L E*.

1. Place the multiplier under the loweſt denomination of the multiplicand.

2. Multiply the number of the loweſt denomination by the multiplier, and find how many ones of the next higher denomination are contained in the product.

3. Write down the exceſs, and carry the ones to the product of the next higher denomination, with which proceed as before; and ſo on, through all the denominations to the higheſt, whoſe product, together with the ſeveral exceſſes, taken as one number, will be the whole amount required.

The method of proof is the ſame as in ſimple multiplication.

EXAMPLES of MONEY.

1. 9 *lb.* of tobacco at 2*s.* 8½*d per lb.*

	2*s.*	8½*d.*
		9
1*l.*	4*s.*	4½. *the anſwer.*

* The product of a number conſiſting of ſeveral parts, or denominations, by any ſimple number whatever, will, evidently, be expreſſed by taking the product of that ſimple number and each part by itſelf as ſo many diſtinct queſtions: thus, 25 *l.* 12 *s.* 6 *d.* multiplied by 9 will be 225 *l.* 108 *s.* 54 *d.* = (by taking the ſhillings from the pence, and the pounds from the ſhillings, and placing them in the ſhillings and pounds reſpectively) 230 *l.* 12 *s.* 6*d.* which is the ſame as the rule; and this will be true when the multiplicand is any compound number whatever.

2. 3*lb.* of green tea at at 9*s.* 6*d.* *per lb.* *Anſ.* 1*l.* 8*s.* 6*d.*
3. 5*lb.* of loaf ſugar at 1*s.* 3*d.* *per lb.* *Anſ.* 6*s.* 3*d.*
4. 9*cwt.* of cheeſe at 1*l.* 11*s.* 5*d.* *per cwt.* *Anſ.* 14*l.* 2*s.* 9*d.*
5. 12 gallons of brandy at 9*s.* 6*d.* *per. gall.* *Anſ.* 5*l.* 14*s.*

Caſe I. If the multiplier exceed 12, multiply ſucceſſively by its component parts, inſtead of the whole number at once, as in ſimple multiplication.

EXAMPLES.

1. 16 *cwt.* of cheeſe at 1*l.* 18*s.* 8*d.* *per cwt.*

1 *l.*	18 *s.*	8 *d.*
		4
7 -	14 -	8
		4
30 *l.*	18 *s.*	8 *d.* *the anſwer.*

2. 28 yards of broad cloth at 19*s.* 4*d.* *per yd.*
Anſ. 27*l.* 1*s.* 4*d.*
3. 35 firkins of butter at 15*s.* $3\frac{1}{2}$ *d.* *per firkin.*
Anſ. 26*l.* 15*s.* $2\frac{1}{2}$*d.*
4. 42 *cwt.* of tallow at 34*s.* 6*d.* *per cwt.*
Anſ. 72 *l.* 9*s.*
5. 64 gallons of brandy at 9*s.* 6*d.* *per gallon.*
Anſ. 30*l.* 8*s.*
6. 96 quarters of rye at 1*l.* 3*s.* 4*d.* *per quarter.*
Anſ. 112*l.*
7. 120 dozen of candles at 5*s.* 9*d.* *per doz.*
Anſ. 34*l.* 10*s.*
8. 132 yards of Iriſh cloth at 2*s.* 4*d.* *per yd.*
Anſ. 15*l.* 8*s.*

9. 144

9. 144 reams of paper at 13*s.* 4*d.* *per ream.*

Anſ. 96 *l.*

10. 1210 yards of ſhalloon at 2*s.* 2*d.* *per yard.*

Anſ. 131*l.* 1*s.* 8*d.*

Caſe II. If the multiplier cannot be produced by the multiplication of ſmall numbers, find the neareſt to it, either greater or leſs, which can be ſo produced; then, multiply by the component parts as before, and for the odd parts, add or ſubtract according as is required.

EXAMPLES.

1. 17 ells of holland at 7*s.* 8½*d.* *per ell.*

	7*s.*	8½ *d.*
		4
1 -	10 -	10
		4
6 -	3 -	4
	7 -	8½
6 *l.*	11 *s.*	0½ *d.* *the anſwer.*

2. 23 ells of dowlas at 1*s.* 6½*d.* *per ell.*

Anſ. 1*l.* 15*s.* 5½*d.*

3. 46 buſhels of wheat at 4*s.* 7¼*d.* *per buſhel.*

Anſ. 10*l.* 11*s.* 9½*d.*

4. 59 yards of tabby at 7*s.* 10*d.* *per yard.*

Anſ. 23*l.* 2*s.* 2*d.*

5. 94 pair of ſilk ſtockings at 12*s.* 2*d.* *per pair.*

Anſ. 57*l.* 3*s.* 8*d.*

6. 117 *cwt.* of malaga raiſins at 1*l.* 2*s.* 3*d.* *per cwt.*

Anſ. 130*l.* 3*s.* 3*d.*

EXAMPLES of WEIGHTS, MEASURES, &c.

lb.	*oz.*	*dwt.*	*gr.*
21	1	7	13
			4

lb.	*oz.*	*dr.*	*sc.*	*gr.*
2	4	2	1	-
				7

cwt.	*qr.*	*lb.*	*oz.*
27	1	13	12
			12

mls.	*fu.*	*pls.*	*yds.*
24	3	20	2
			6

yds.	*qrs.*	*na.*
127	2	2
		8

ac.	*ro.*	*po.*
27	2	1
		9

tuns.	*hhd.*	*gal.*	*pts.*
29	1	20	3
			5

we.	*qr.*	*bu.*	*pe.*
27	1	7	2
			7

mo.	*we.*	*da.*	*ho.*	*min.*
175	3	6	20	59
				11

COMPOUND DIVISION.

Compound Division teacheth to find how often one given number is contained in another of different denominations.

RULE.*

1. Place the numbers as in simple division.

2. Be-

* To divide a number consisting of several denominations by any simple number whatever, is, evidently, the same as dividing all the parts or members of which that number is composed by the same simple

2. Begin at the left hand, and divide each denomination by the divisor, setting the quotients under their respective dividends.

3. But if there be a remainder, after dividing any of the denominations except the least, find how many of the next lower denomination it is equal to, and add it to the number, if any, which was in this denomination before; then divide the sum as usual, and so on till the whole is finished.

The method of proof is the same as in simple division.

EXAMPLES of MONEY.

1. Divide 225*l.* 2*s.* 4*d.* by 2.

2)225*l.*	2*s.*	4*d.*	
112*l.*	11*s.*	2*d.*	*the quotient.*

2. Divide	751*l.*	14*s.*	7$\frac{3}{4}$*d.*	by	3.	*Ans.*	250*l.*	11*s.*	6$\frac{1}{2}$*d.*
3. Divide	821*l.*	17*s.*	9$\frac{3}{4}$*d.*	by	4.	*Ans.*	205*l.*	9*s.*	5$\frac{1}{4}$*d.*
4. Divide	2382*l.*	13*s.*	5$\frac{1}{2}$*d.*	by	5.	*Ans.*	476*l.*	10*s.*	8$\frac{1}{4}$*d.*
5. Divide	28*l.*	2*s.*	1$\frac{1}{2}$*d.*	by	6.	*Ans.*	4*l.*	13*s.*	8$\frac{1}{4}$*d.*
6. Divide	55*l.*	14*s.*	–$\frac{3}{4}$*d.*	by	7.	*Ans.*	7*l.*	19*s.*	1$\frac{3}{4}$*d.*
7. Divide	6*l.*	5*s.*	4 *d.*	by	8.	*Ans.*		15*s.*	8 *d.*
8. Divide	135*l.*	10*s.*	7 *d.*	by	9.	*Ans.*	15*l.*	1*s.*	2 *d.*
9. Divide	21*l.*	18*s.*	4 *d.*	by	10.	*Ans.*	2*l.*	3*s.*	10 *d.*
10. Divide	227*l.*	10*s.*	5 *d.*	by	11.	*Ans.*	20*l.*	13*s.*	8 *d.*
11. Divide	1332*l.*	11*s.*	8$\frac{1}{2}$*d.*	by	12.	*Ans.*	111*l.*	0*s.*	11$\frac{1}{2}$*d.*

ple number. And this will be true when any of the parts are not an exact multiple of the divisor: for by conceiving the number, by which it exceeds that multiple, to have its proper value by being placed in the next lower denomination, the dividend will still be divided into parts, and the true quotient found as before: thus, 25*l.* 12*s.* 3*d.* divided by 9, will be the same as 18*l.* 144*s.* 99*d.* divided by 9, which is equal to 2*l.* 16*s.* 11*d.* as by the rule; and the method of carrying from one denomination to another is exactly the same.

Case I. If the divisor exceed 12, divide continually by its component parts, as in simple division.

EXAMPLES.

1. What is cheese *per cwt.* if 16 *cwt.* cost 30*l.* 18*s.* 8*d.*?

4)30 *l.* 18 *s.* 8 *d.*

4)7 *l.* 14 *s.* 8 *d.*

1 *l.* 18 *s.* 8 *d.* *the answer.*

2. If 20 *cwt.* of tobacco comes to 120*l.* 10*s.* what is that *per cwt.*? *Ans.* 6*l.* 0*s.* 6*d.*
3. Divide 57*l.* 3*s.* 7*d.* by 35. *Ans.* 1*l.* 12*s.* 8*d.*
4. Divide 85*l.* 6*s.* by 72. *Ans.* 1*l.* 3*s.* 8$\frac{1}{4}$*d.*
5. Divide 31*l.* 2*s.* 10$\frac{1}{2}$*d.* by 99. *Ans.* 6*s.* 3$\frac{1}{2}$*d.*
6. At 18*l.* 18*s.* *per cwt.* how much *per lb.*? *Ans.* 3*s.* 4$\frac{1}{2}$*d.*

Case II. If the divisor cannot be produced by the multiplication of small numbers, divide by it after the manner of long division.

EXAMPLES.

1. Divide 74*l.* 13*s.* 6*d.* by 17.

```
17) 74 l. 13 s. 6 d. (4 l. 7 s. 10 d. the quotient.
    68
    --
     6
    20
    --
   133
   119
   ---
    14
    12
   ---
   174
    17
    --
     4
```

2. Divide

2. Divide 23*l.* 15*s.* 7¼*d.* by 37. *Ans.* 12*s.* 10¾*d.*
3. Divide 199*l.* 3*s.* 10*d.* by 53. *Ans.* 3*l.* 15*s.* 2*d.*
4. Divide 675*l.* 12*s.* 6*d.* by 138. *Ans.* 4*l.* 17*s.* 11*d.*
5. Divide 315*l.* 3*s.* 10¼*d.* by 365. *Ans.* 17*s.* 3¼*d.*

EXAMPLES *of* WEIGHTS *and* MEASURES.

1. Divide 23 *lb.* 7 *oz.* 6 *dwts.* 12 *grs.* by 7.
Ans. 3 *lb.* 4 *oz.* 9 *dwts.* 12 *grs.*
2. Divide 13 *lb.* 1 *oz.* 2 *dr.* - *scr.* 10 *gr.* by 12.
Ans. 1 *lb.* 1 *oz.* 0 *dr.* 2 *scr.* 10 *gr.*
3. Divide 1061 *cwt.* 2 *qr.* by 28.
Ans. 37 *cwt.* 3 *qrs.* 18 *lb.*
4. Divide 375 *mi.* 2 *fur.* 7 *po.* 2 *yds.* 1 *fe.* 2 *in.* by 39.
Ans. 9 *mi.* 4 *fur.* 39 *po.* - *yds.* 2 *fe.* 8 *in.*
5. Divide 571 *yds.* 2 *qrs.* 1 *na.* by 47.
Ans. 12 *yds.* - *qrs.* 2 *na.*
6. Divide 51 *ac.* 2 *ro.* 3 *po.* by 51.
Ans. 1 *ac.* - *ro.* 1 *po.*
7. Divide 10 *tu.* 2 *hhds.* 17 *gall.* 2 *pi.* by 67.
Ans. 39 *galls.* 6 *pi.*
8. Divide 120 *la.* 2 *qrs.* 1 *bu.* 2 *pe.* by 74.
Ans. 1 *la.* 6 *qrs.* 1 *bu.* 3 *pe.*
9. Divide 120 *mo.* 2 *we.* 3 *da.* 5 *ho.* 20 *min.* by 111.
Ans. 1 *mo.* 0 *we.* 2 *da.* 10 *ho.* 12 *mi.*

REDUCTION.

Reduction is the method of bringing numbers from one name or denomination to another, so as still to retain the same value.

RULE*.

I. *When the reduction is from a greater name to a less.*

* The reason of this rule is exceedingly obvious; for pounds are brought into shillings by multiplying them by 20; shillings into pence by multiplying them by 12; and pence into farthings by multiplying

Multiply the highest name or denomination by as many as make one of the next less, adding to the product the parts of the second name; then multiply this sum by as many as make one of the next less name, adding to the product the parts of the third name; and so on, through all the denominations to the last.

II. *When the Reduction is from a less name to a greater.*

Divide the given number by as many as make one of the next superior denomination; and this quotient again by as many as make one of the next following; and so on through all the denominations to the highest; and this last quotient, together with the several remainders, will be the answer required.

The method of proof is by reversing the question.

EXAMPLES.

1. In 1465*l.* 14*s.* 5*d.* how many farthings?

20	4)1407092
29314	12)351773
12	2,0)2931,4—5
351773	1465*l.* 14*s.* 5*d.* *proof.*
4	
1407092 *the answer.*	

2. In 12*l.* how many farthings? *Ans.* 11520.

3. In 6169 pence how many pounds? *Ans.* 25*l.* 14*s.* 1*d.*

---

tiplying them by 4; and the contrary by division: and this will be true in the reduction of numbers consisting of any denominations whatsoever.

5. In

4. In 35 guineas how many farthings? *Ans.* 35280.

5. In 420 quarter-guineas how many moidores? *Ans.* 81 *and* 18*s.*

6. In 231*l.* 16*s.* how many ducats at 4*s.* 9*d.* each? *Ans.* 976.

7. In 274 marks each 13*s.* 4*d.* and 87 nobles each 6*s.* 8*d.* how many pounds? *Ans.* 211*l.* 13*s.* 4*d.*

8. In 1776 quarter-guineas how many six-pences? *Ans.* 18648.

9. Reduce 1776 six-and-thirties to half crowns? *Ans.* 25574$\frac{2}{5}$.

10. In 50807 moidores how many pieces of coin each 4*s.* 6*d*? *Ans.* 304842.

11. In 213210 grains how many *lb*? *Ans.* 37 *lb.* 9*gr.*

12. In 59*lb.* 13*dwts.* 5*gr.* how many grains? *Ans.* 340157.

13. In 8012131 grains how many *lb*? *Ans.* 1390*lb.* 11*oz.* 18*dwts.* 19*gr.*

14. In 35 *ton.* 17 *cwt.* 1 *qr.* 23*lb.* 7*oz.* 13 *dr.* how many drams? *Ans.* 20571005.

15. In 37 *cwt.* 2 *qr.* 17 *lb.* how many *lb.* troy, *a lb.* avoirdupois being equal to 14 *oz.* 11 *dwts.* 15$\frac{1}{2}$*gr.* troy? *Ans.* 5124*lb.* 5*oz.* 10*dwt.* 11$\frac{1}{2}$*gr.*

16. How many barley corns will reach round the world, supposing it, according to the best calculations, to be 8340 leagues? *Ans.* 4755801600

17. In 17 pieces of cloth each 27 flemish ells, how many yards? *Ans.* 344 *yds.* 1 *qr.*

18. How many minutes are there since the birth of Christ to this present year 1776, allowing the year to consist of 365 *da.* 5 *ho.* 48 *min.* 58 *sec.*? *Ans.* 934085364 *m.* 8 *sec.*

THE

THE RULE OF THREE DIRECT.

The Rule of Three direct teacheth, by having three numbers given to find a fourth, that ſhall have the ſame proportion to the third as the ſecond has to the firſt.

RULE*.

1. State the queſtion; that is, place the numbers ſo, that the firſt and third may be the terms of ſuppoſition and demand, and the ſecond of the ſame kind with the anſwer required.

2. Bring

* This Rule, on account of its great and extenſive uſefulneſs, is oftentimes called THE GOLDEN RULE OF PROPORTION: for, on a proper Application of it, and the preceding rules, the whole buſineſs of arithmetic, as well as every mathematical enquiry, depends. The rule itſelf is founded on this obvious principle, that the magnitude or quantity of any effect varies conſtantly in proportion to the varying part of the cauſe: thus, the quantity of goods bought is in proportion to the money laid out; the ſpace gone over by an uniform motion is in proportion to the time, &c.—As the idea annexed to the term proportion is eaſily conceived, it would be more perplexing than inſtructive to explain, in this place, what is meant by it, in a ſtrict geometrical ſenſe. It may be ſufficient, therefore, to obſerve, that independant of the preciſe meaning of that word, and its deducible properties, the truth of the rule, as applied to ordinary enquiries, may be made very evident, by attending only to principles already explained.—It is ſhewn in multiplication of money, that the price of one multiplied by the quantity is the price of the whole; and in diviſion, that the price of the whole divided by the quantity is the price of one. Now, in all caſes of valuing goods, &c. where one is the firſt term of the proportion, it is plain that the anſwer found by this rule will be the ſame as that found by multiplication of money; and where one is the laſt term of the proportion it will be the ſame as that found by diviſion of money. In like manner, if the firſt term be any number whatever, it is plain that the product of the ſecond and third terms will be greater than the true anſwer required

by

2. Bring the firſt and third numbers into the ſame denomination, and the ſecond into the loweſt name mentioned.

3. Multiply the ſecond and third numbers together, and divide the product by the firſt, and the quotient will be the anſwer to the queſtion, in the ſame denomination you left the ſecond number in; which may be brought into any other denomination required.

Two or more ſtatings are ſometimes neceſſary, which may always be known from the nature of the queſtion.

The method of proof is by inverting the queſtion.

Ex-

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by as much as the price in the ſecond term exceeds the price of one, or as the firſt terms exceeds an unit. Conſequently this product divided by the firſt term will give the true anſwer required, and is the rule.

Direct and inverſe proportion are properly only parts of the ſame general rule, and, in a ſcientific arrangement, it would be beſt to conſider them in that manner; but I have here preſerved the common diſtinctions, and contented myſelf with looſe definitions, becauſe I have obſerved that young perſons in general find them more intelligible.

Note 1. When it can be done, multiply and divide as in compound multiplication and diviſion.

2. If the 1ſt. term, and either the 2d. or 3d. can be divided by any number without a remainder, let them be divided, and the quotients uſed inſtead of them.

The four following methods of operation, when they can be uſed perform the work in a much ſhorter manner than the general rule.

1. Divide the 2d. term by the 1ſt. and multiply the quotient into the 3d. and the product will be the anſwer.

2. Divide the 3d. term by the 1ſt. and multiply the quotient into the 2d. and the product will be the anſwer.

3. Divide the 1ſt. term by the 2d. and the 3d. by that quotient, and the laſt quotient will be the anſwer.

4. Divide the 1ſt. term by the 3d. and the ſecond by that quotient, and the laſt quotient will be the anſwer.

There will ſometimes be a difficulty in ſeparating the parts of complicated queſtions, where two or more ſtatings are required, and in preparing the queſtion for ſtating, or after a proportion is wrought; but as there can be no general directions given for the management of theſe caſes, it muſt be left to the judgement and experience of the learner.

Examples.

1. If 24 *lb.* of raisins cost 6*s.* 6*d.* what will 18 frails cost, each weighing neat 3 *qrs.* 18 *lb.* ?

If 24*lb.* : 6*s.* 6*d.* :: 18 *frails each* 3 *qrs.* 18*lb.*

```
           12                        28
           ──                        ──
           78                        102
                                      18
                                     ───
                                     816
                                     102
                                     ───
                                    1836
                                      78
                                   ─────
                                   14688
                                   12852
                                  ──────      (12)
                               24)143208    (5967
                                   232      ─────
                                   160   2,0)49,7—3
                                   168      ─────────
                                   ...      £. 24-17-3
                                   ───
```

Answer 24*l.* 17*s.* 3*d.*

2. What is the value of a *cwt.* of sugar at $5\frac{1}{2}$*d per lb.* *Ans.* 2*l.* 11*s.* 4*d.*

3. What is the value of a chaldron of coals at $11\frac{1}{2}$*d per bushel?* *Ans.* 1*l.* 14*s.* 6*d.*

4. At $10\frac{1}{2}$*d. per lb.* what is the value of a firkin of butter containing 56 *lb.* ? *Ans.* 2*l.* 9*s.*

5. What is the value of a pipe of wine at $10\frac{1}{2}$*d. per pint.* ? *Ans.* 44*l.* 2*s.*

6. At 3*l.* 9*s.* *per cwt.* what is the value of a pack of wool weighing 2*cwt.* 2*qrs.* 13*lb.* ? *Ans.* 9*l.* 0*s.* 6*d.* $\frac{12}{112}$

7. What

7. What is the value of $1\frac{1}{2}$ *cwt* of coffee at $5\frac{1}{2}d.$ *per oz.*? *Ans.* 61*l.* 12*s.*
8. What is the value of $19\frac{1}{2}$ chaldron of coals at 1 *l.* 11*s.* 6*d.* *per* chaldron? *Ans.* 30*l.* 14*s.* 3*d.*
9. Bought 3 casks of raisins each weighing 2 *cwt.* 2 *qrs.* 25 *lb.* what will they come to at 2*l.* 1*s.* 8*d.* *per cwt*? *Ans.* 17*l.* 0*s.* $4\frac{3}{4}d.\frac{32}{112}$
10. What is the value of 2 *qrs.* 1 *na.* of velvet at 19 *s.* $8\frac{1}{2}d.$ *per eng. ell.*? *Ans.* 8*s.* $10\frac{1}{4}d.\frac{14}{20}$
11. Bought 12 pockets of hops, each weighing 1 *cwt.* 2 *qrs.* 17 *lb*; what do they come to at 4*l.* 1*s.* 4*d.* *per cwt.*? *Ans.* 80*l.* 12*s.* $1\frac{1}{2}d.\frac{96}{112}$
12. What is the tax upon 745*l.* 14*s.* 8*d.* at 3*s.* 6*d.* in the pound? *Ans.* 130*l.* 10*s.* $0\frac{1}{4}d.\frac{48}{240}$
13. If $\frac{3}{4}$ of a yard of velvet cost 7*s.* 3*d.* how many yards can I buy for 13*l.* 15*s.* 6*d*? *Ans.* $28\frac{1}{2}$ *yards.*
14. If an ingot of gold weighing 9 *lb.* 9 *oz.* 12 *dwts.* be worth 411 *l.* 12 *s.* what is that *per grain*? *Ans.* $1\frac{3}{4}d.$
15. How many quarters of corn can I buy for 40 guineas at 4 *s.* *per bushel*? *Ans.* 26 *qrs.* 2 *bu.*
16. If 1 *eng. ell.* 2 *qrs.* cost 4*s.* 7*d.* what will $39\frac{1}{2}$ yards cost? *Ans.* 5*l.* 3*s.* $5\frac{1}{4}d.\frac{5}{7}$
17. What is the value of a pack of wool weighing 2 *cwt.* 1 *qr.* 19 *lb.* at 8*s.* 6*d.* *per stone*? *Ans.* 8*l.* 4*s.* $6\frac{1}{4}d.\frac{10}{14}$
18. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards at 16 *l.* 4 *s.* *per* piece, what is the value of the whole, and the rate *per yard*? *Ans.* 388 *l.* 16 *s.* *at* 12 *s.* *per yard.*
19. If an ounce of silver be worth 5*s.* 6*d.* what is the price of a tankard that weighs 1 *lb.* 10 *oz.* 10 *dwts.* 4 *grs.*? *Ans.* 6 *l.* 3 *s.* $9\frac{1}{2}$ $d.\frac{96}{480}$
20. What does 59 *cwt.* 2 *qrs.* 24 *lb.* of tobacco come to at 2 *l.* 14 *s.* 5 *d.* *per cwt.*? *Ans.* 162*l.* 9*s.* 5*d.*$\frac{48}{112}$
21. What is the half year's rent of 547 acres of land, at 15 *s.* 6 *d.* *per acre*? *Ans.* 211 *l.* 19 *s.* 3 *d.*
22. At half a guinea *per* week, how many months board can I have for 100 *l.*? *Ans.* 47 *mo.* 2 *we.*$\frac{60}{126}$

23. Bought

23. Bought 1000 *flem. ells* of cloth for 90*l.* how must I sell it *per* ell in *London* to gain 10*l.* by the whole?

Ans. 3*s.* 4*d.*

24. Suppose a gentleman's income is 500 guineas a year, and he spends 19*s.* 7*d. per* day, one day with another, how much will he have saved at the year's end?

Ans. 167*l.* 12*s.* 1*d.*

25. If $1\frac{3}{4}$ ounce of silver plate cost 10*s.* $11\frac{1}{4}$*d.* what will a service, weighing 327 *oz.* 12 *dwts.* 9 *gr.* cost at that rate? *Ans.* 102*l.* 7*s.* $7\frac{1}{4}$*d.* $\frac{525}{840}$

26. At 13*s.* $2\frac{1}{2}$*d. per* yard, what is the value of a piece of cloth containing $52\frac{3}{4}$ *eng. ells*?

Ans. 43*l.* 10*s.* $11\frac{1}{4}$*d.* $\frac{6}{16}$

27. How many *eng. ells* of holland may be bought for 100 guineas at 8*s.* $9\frac{1}{2}$*d. per* yard?

Ans. 191 *ells.* 0 *qr.* $\frac{190}{422}$

28. What is the value of 172 pigs of lead each weighing 3 *cwt.* 2 *qrs.* $17\frac{1}{2}$*lb.* at 8*l.* 17*s.* 6*d. per* fother of $19\frac{1}{2}$ *cwt.*? *Ans.* 286*l.* 4*s.* $4\frac{1}{2}$*d.*

29. Bought 25 pieces of holland, each containing 25 *eng. ells*, for 300 guineas, what is that *per* yard?

Ans. 8*s.* $0\frac{3}{4}$*d.* $\frac{225}{3125}$.

30. If I buy 15 yards of cloth for 11 guineas, how many flemish ells can I buy for 240*l.* 13*s.* 4*d.* at the same rate? *Ans.* 416 *flem. ells* $\frac{1516}{2772}$

31. The rents of a whole parish amount to 1750*l.* and a rate is granted of 32*l.* 16*s.* 6*d.*; what is that in the pound? *Ans.* $4\frac{1}{2}$*d.* $\frac{2880}{420000}$

32. If my horse stands me in $11\frac{1}{2}$*d. per* day keeping, what will be the charge of 11 horses for the year?

Ans. 192*l.* 7*s.* $8\frac{1}{2}$*d.*

33. A person breaking owes in all 1490*l.* 5*s.* 10*d.* and has in money, goods and recoverable debts 784*l.* 17*s.* 4*d.*: if these things are delivered to his creditors what will they get in the pound? *Ans.* 10*s.* $6\frac{1}{4}$*d.* $\frac{20993}{357670}$

34. What must 40*s.* pay towards a tax, when 652*l.* 13*s.* 4*d.* is assessed at 83*l.* 12*s.* 4*d.*? *Ans.* 5*s.* $1\frac{1}{4}$*d.* $\frac{15376}{15664}$

35. Bought

35. Bought 3 tons of oil for 151 *l.* 14 *s.* 85 gallons of which being damaged, I desire to know how I may sell the remainder *per gallon* so as neither to gain or lose by the bargain? *Ans.* 4 *s.* $6\frac{1}{4}$ *d.* $\frac{25}{671}$

36. What quantity of water must I add to a pipe of mountain wine value 33 *l.* to reduce the first cost to 4 *s.* 6 *d. per gallon?* *Ans.* $20\frac{2}{3}$ *gallons.*

37. If 15 ells of stuff $\frac{3}{4}$ yard wide cost 37 *s.* 6 *d.* what what will 40 ells of the same stuff cost, being yard wide? *Ans.* 6 *l.* 13 *s.* 4 *d.*

38. Shipped for Barbadoes 500 pair of stockings at 3 *s.* 6 *d. per* pair, and 1650 yards of baize at 1 *s.* 3 *d. per yard*, and have received in return 348 gallons of rum at 6 *s.* 8 *d. per* gallon, and 750 *lb.* of indigo at 1 *s.* 4 *d. per lb.*: what remains due upon my adventure? *Ans.* 24 *l.* 12 *s.* 6 *d.*

THE RULE OF THREE INVERSE.

The Rule of Three Inverse teacheth by having three numbers given to find a fourth, that shall have the same proportion to the second as the first has to the third.

If *more* require *more*, or *less* require *less*, the question belongs to the rule of three direct.

But if *more* require *less*, or *less* require *more*, it belongs to the rule of three inverse. *

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* *More* requiring *more* is when the third term is greater than the first, and requires the fourth term to be greater than the second.

And *less* requiring *less* is when the third term is less than the first, and requires the fourth term to be less than the second.

In like manner, *more* requiring *less* is when the third term is greater than the first, and requires the fourth term to be less than the second,

And *less* requiring *more* is when the third term is less than the first, and requires the fourth term to be greater than the second.

RULE.

R U L E.*

1. State and reduce the terms as in the rule of three direct.

2. Multiply the first and second terms together, and divide their product by the third, and the quotient is the answer to the question, in the same denomination you left the second number in.

The method of proof is by inverting the question.

EXAMPLES.

1. What quantity of shalloon that is 3 quarters of a yard wide, will line $7\frac{1}{2}$ yards of cloth, that is $1\frac{1}{2}$ yards wide?

```
1 yd. 2 qrs.   :   7 yds. 2 qrs.   ::   3 qrs.
   4                  4
  ---                ---
   6                 30
                      6
                    ----
                  3)180
                    ----
                  4)60
                    ---
                    15 yards, the answer.
```

2. If 100 workmen can finish a piece of work in 12 days, how many are sufficient to do the same in 3 days? *Ans.* 400 *men.*

---

* The reason of this rule may be explained from the principles of compound multiplication and division, in the same manner as the direct rule. *For example.* If 6 men can do a piece of work in 10 days, in how many days will 12 men do it?

As 6 *men* : 10 *days* :: 12 *men* : $\frac{6 \times 10}{12} = 5$ *days*, the answer. And here the product of the first and second terms *i. e.* 6 times 10, or 60, is evidently the time in which one man would perform the work; therefore 12 men will do it in one twelfth part of that time, or 5 days; and this reasoning is applicable to any other instance whatever.

3. How much in length that is $4\frac{1}{2}$ inches broad will make a square foot? *Ans.* 32 *inches.*

4. How many yards of matting 2 *fe.* 6 *in.* broad will cover a floor that is 27 *fe.* long and 20 *fe.* broad? *Ans.* 72 *yards.*

5. How many yards of cloth 3 *qrs.* wide are equal in measure to 30 *yds.* 5 *qrs.* wide? *Ans.* 50 *yards.*

6. A. borrowed of his friend B. 250*l.* for 7 months, promising to do him the like kindness: some time after B. had occasion for 300*l.* how long may he keep it to be made full amends for the favour? *Ans.* 5 *mo.* and 25 *days.*

7. If, when the price of a bushel of wheat is 6 *s.* 3 *d.* the penny loaf weigh 9 *oz.* what ought it to weigh when wheat is at 8 *s.* $2\frac{1}{2}$ *d. per bushel?* *Ans.* 6 *oz.* 13 *dr.*

8. How many yards of stuff 3 *qrs.* broad will line a cloak that is $5\frac{1}{2}$ *yds.* in length and $1\frac{1}{4}$ *yd.* broad? *Ans.* 9 *yds.* $\frac{1}{6}$

9. If $4\frac{1}{2}$ *cwt.* may be carried 36 miles for 35 *s.* how many pounds can I have carried 20 miles for the same money. *Ans.* 907 *lb* $\frac{4}{20}$

10. How much in length that is $13\frac{1}{2}$ poles in breadth must be taken to contain an acre? *Ans.* 11 *po.* 14 *fe.* $\frac{99}{1782}$

11. How many yards of canvas that is ell wide, will line 20 yards of say that is 3 *qrs.* wide? *Ans.* 12 *yds.*

12. If 30 men can perform a piece of work in 11 days; how many men will accomplish another piece of work four times as big in a fifth part of the time? *Ans.* 600.

13. A wall that is to be built to the height of 27 feet, was raised 9 feet by 12 men in 6 days: how many men must be employed to finish the wall in 4 days at the same rate of working? *Ans.* 36 *men.*

COM-

COMPOUND PROPORTION.

Compound Proportion teacheth to resolve such questions as require two or more statings by simple proportion; and that, whether they are direct or inverse.

RULE.*

1. Let that term be put in the second place which is of the same denomination with the term sought.

2. Place the terms of supposition, one above another, in the first place; and the terms of demand, one above another, in the third place.

3. The first and third term of every row will be of one name, and must be reduced to the same denomination.

4. Examine every row separately: by saying, if the first term give the second, does the third require more or less? if it require *more* mark the *less* extreme with a cross; but if *less* mark the *greater* extreme.

5. Multiply all those numbers together which are marked for a divisor, and those which are not marked for a dividend, and the quotient will be the answer sought.

Note, when the same numbers are found in the divisor as in the dividend they may be thrown out of both. Or

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* The reason of this rule may be readily shown from the nature of direct and inverse proportion: for every row in this case is a particular stating in one of those rules; and therefore if all the separate dividends be collected together into one dividend, and all the divisors into one divisor, their quotient must be the answer sought. Thus, in example the first.

As 9 bush. : 16 horses :: 24 bush. : $\frac{24 \times 16}{9}$ by rule of three direct.

As 6 days : $\frac{24 \times 16}{9}$ horses :: 7 days $\frac{24 \times 16 \times 6}{9 \times 7}$ by rule of three inverse, which is the same as the rule.

any

any numbers may be divided by their greatest common divisor, and the quotients taken instead of them.

EXAMPLES.

1. If 16 horses can eat up 9 bushels of oats in 6 days, how many horses would eat up 24 bushels in 7 days, at the same rate?

$$\begin{array}{lcccc} +\ 9\ \textit{bushels} & : & 16\ \textit{horses} & :: & 24\ \textit{bushels.} \\ \ \ 6\ \textit{days} & : & \overline{} & :: & 7\ \textit{days}\ + \end{array}$$

$$\frac{6\times16\times24}{9\times7},\ \textit{by contraction} = \frac{2\times16\times24}{3\times7} = \frac{2\times16\times8}{1\times7}$$

$$= \frac{256}{7} = 36\tfrac{4}{7}\ \textit{horses, the answer.}$$

2. If a family of 9 people spend 120*l.* in 8 months, how much will serve a family of 24 people 16 months?
Ans. 1066 *l.* 13 *s.* 4 *d.*

3. If 8 men can dig 24 yards of earth in 6 days; how many men must there be to dig 18 yards in 3 days?
Ans. 12 *men.*

4. If 2 men can do 12 $\frac{3}{4}$ rods of ditching in $6\frac{1}{2}$ days; how many rods may be done by 18 men in 14 days?
Ans. $247\frac{2}{13}$ *rods.*

5. If a regiment of soldiers, consisting of 939 men, can eat up 351 quarters of wheat in 7 months; how many soldiers will eat up 1464 quarters in 5 months at that rate?
Ans. $5483\frac{28}{195}$

6. If the carriage of 5 *cwt.* 3 *qr.* 150 miles, cost 3*l.* 7*s.* 4*d.* what must be paid for the carriage of 7 *cwt.* 2 *qr.* 25 *lb.* 64 miles at the same rate?
Ans. 1 *l.* 18 *s.* 7 *d.* $\frac{127}{2415}$

7. If 248 men, in 5 days of 11 hours each, dig a trench 230 yards long, 3 wide and 2 deep, in how many days, of 9 hours long, will 24 men dig a trench of 420 yards long, 5 wide and three deep?
Ans. $288\frac{59}{207}$ *days.*

PRAC-

PRACTICE.

Practice is a contraction of the rule of three direct, when the first term happens to be an unit, or one; and has its name from its daily use amongst merchants and tradesmen, being an easy and concise method of working most questions that occur in trade and business.

The method of proof is by the rule of three direct.

An aliquot part of any number, is such a part of it, as being taken a certain number of times, doth exactly make that number.

CASE. 1*

When the price is less than a penny.

RULE.

Divide by the aliquot parts of a penny, and then by 12 and by 20, and it will give the answer required.

Divide

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* As most of the following compendiums are only particular cases of a more general rule, it will be sufficient, for their illustration, to explain the principles on which the rule itself is founded.

General Rule. 1. Suppose the price of the given quantity to be 1*l.* or 1*s.* as is most convenient; then will the quantity itself be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each, will be the true answer required.

EXAMPLE.

What is the value of 526 yards of cloth, at 3*s.* 10¼*d.* *per* yard.

		526			*Ans.*	at	1*l.*			
3*s.* 4*d.* is	$\frac{1}{6}$	87	13	4	ditto	at	0	3	4	
4*d.* is	$\frac{1}{10}$	8	15	4	ditto	at	0	0	4	
2*d.* is	$\frac{1}{2}$	4	7	8	ditto	at	0	0	2	
$\frac{1}{4}$ is	$\frac{1}{8}$	0	10	11½	ditto	at	0	0	0¼	
		101	7	3½	ditto	at	0	3	10¼	the full price.

In

EXAMPLES.

$$\begin{array}{r}
4506 \text{ at } \frac{3}{4} \\
\hline
\frac{1}{2} \text{ is } \frac{1}{2}\ 2253 \\
\frac{1}{4} \text{ is } \frac{1}{2}\ 1126\frac{1}{2} \\
\hline
12)3379\frac{1}{2} \\
\hline
2,0)28,1 : 7 \\
\hline
\end{array}$$

14*l.* 1*s.* $7\frac{1}{2}$*d.* *the anſwer.*

3456 at $\frac{1}{4}$. *Anſ.* 3*l.* 12*s.* 347 at $\frac{1}{2}$. *Anſ.* 14*s.* $5\frac{1}{2}$*d.*
846 at $\frac{3}{4}$. *Anſ.* 2*l.* 12*s.* $10\frac{1}{2}$*d.* 810 at $\frac{3}{4}$. *Anſ.* 2*l.* 10*s.* $7\frac{1}{2}$*d.*

CASE 2.

When the price is an aliquot part of a ſhilling.

RULE.

Divide the given number by the aliquot part, and the quotient is the anſwer in ſhillings, which reduce into pounds as before.

In the above example, it is plain, that the quantity 526 is the anſwer at 1*l.* conſequently, as 3*s.* 4*d.* is the $\frac{1}{6}$ of a pound, $\frac{1}{6}$ part of that quantity or 87*l.* 13*s.* 4*d.* is the price at 3*s.* 4*d.* In like manner, as 4*d.* is the $\frac{1}{10}$ part of 3*s.* 4*d.* ſo $\frac{1}{10}$ of 87*l.* 13*s.* 4*d.* or 8*l.* 15*s.* 4*d.* is the anſwer at 4*d.* And by reaſoning in this way 4*l.* 7*s.* 8*d.* will be ſhewn to be the price at 2*d.* and 10*s.* $11\frac{1}{2}$*d.* the price at $\frac{1}{4}$. Now as the ſum of all theſe parts is equal to the whole price, (3*s.* $10\frac{1}{4}$*d.*) ſo the ſum of the anſwers belonging to each price will be the anſwer at the full price required. And the ſame will be true in any example whatever.

EXAMPLES.

3 *d.* *is* ¼ 1728 *at* 3*d.*

2,0) 43,2

21 *l.* 12 *s.* *the answer.*

437 at 1*d.*	*Ans.* 1*l.* 16*s.* 5*d.*	352 at 1½*d.*	*Ans.* 2*l.* 4*s.*
5275 at 2*d.*	*Ans.* 43*l.* 19*s.* 2*d.*	1776 at 3 *d.*	*Ans.* 22*l.* 4*s.*
6771 at 4*d.*	*Ans.* 112*l.* 17*s.*	899 at 6*d.*	*Ans.* 22*l.* 9*s.* 6*d.*

CASE. 3.

When the price is pence and farthings, and is no aliquot part of a shilling.

RULE.

Divide the given number by some aliquot part of a shilling, and then consider what part of the said aliquot part the rest is, and divide the quotient thereby; and the last quotient, together with the former, will be the answer in shillings, which reduce into pounds as before.

EXAMPLES.

876 *at* 8½*d.*

6*d.* *is* ½ 438
2*d.* *is* ⅓ 146
½*d.* *is* ¼ 36 - 6

2,0) 62,0 - 6

31 *l.* - 0 *s.* 6 *d.* *the answer.*

372 at 1¾*d.*	*Ans.* 2 *l.*	14 *s.*	3 *d.*
325 at 2¼*d.*	*Ans.* 3 *l.*	0 *s.*	11¼ *d.*
827 at 4½*d.*	*Ans.* 15 *l.*	10 *s.*	1½ *d.*
2700 at 7¼*d.*	*Ans.* 81 *l.*	11 *s.*	3 *d.*
2150 at 9¾*d.*	*Ans.* 87 *l.*	6 *s.*	10½ *d.*
1720 at 11½*d.*	*Ans.* 82 *l.*	8 *s.*	4 *d.*

CASE 4.

When the price is any number of shillings under 20.

RULE.

1. *When the price is an even number*, multiply the given number by ½ of it, doubling the first figure to the right hand for shillings, and the rest are pounds.

2. *When the price is an odd number*, find for the greatest even number as before, to which add $\frac{1}{20}$ of the given number for the odd shilling, and the sum is the answer.

EXAMPLES.

	243		*at* 4*s.*
	2		
	48 *l.*	12 *s.*	*the answer.*

	566		*at* 7*s.*
	3		
	169	- 16	
1 *s.* *is* $\frac{1}{20}$	28	- 6	
	198 *l.* -	2 *s.*	*the answer.*

2757	at	1 *s.*	*Ans.*	137 *l.*	17 *s.*
2643	at	2 *s.*	*Ans.*	264 *l.*	6 *s.*
3271	at	5 *s.*	*Ans.*	817 *l.*	15 *s.*
872	at	8 *s.*	*Ans.*	348 *l.*	16 *s.*
372	at	11 *s.*	*Ans.*	204 *l.*	12 *s.*
5271	at	14 *s.*	*Ans.*	3689 *l.*	14 *s.*
3142	at	17 *s.*	*Ans.*	2670 *l.*	14 *s.*
264	at	19 *s.*	*Ans.*	250 *l.*	16 *s.*

CASE 5.

When the price is shillings and pence, which make some aliquot part of a pound.

RULE.

Divide the given quantity by the aliquot part, and the quotient is the anſwer in pounds.

EXAMPLES.

3 *s.* 4 *d.* is $\frac{1}{6}$	329	*at* 3 *s.* 4 *d.*
	54 *l.* 16 *s.* 8 *d.*	*the anſwer.*

7150 at 1 *s.* 8 *d.*	*Anſ.* 595 *l.* 16 *s.* 8 *d.*
2715 at 2 *s.* 6 *d.*	*Anſ.* 339 *l.* 7 *s.* 6 *d.*
3150 at 3 *s.* 4 *d.*	*Anſ.* 525 *l.* 0 *s.* 0 *d.*
2710 at 6 *s.* 8 *d.*	*Anſ.* 903 *l.* 6 *s.* 8 *d.*

CASE 6.

When the price is ſhillings and pence which make no aliquot part of a pound.

RULE.

Bring out the anſwer the ſhorteſt way that can be done, either by working for an even number of ſhillings and other aliquot parts, or by dividing the price into ſeveral parts, either of the given number, or of one another.

EXAMPLES.

	765	*at* 5 *s.* 9 *d.*		
5 *s.* is $\frac{1}{4}$	191	- 5		
6 *d.* is $\frac{1}{10}$	19	- 2	- 6	
3 *d.* is $\frac{1}{2}$	9	- 11	- 3	
	209 *l.*	- 18 *s.*	- 9 *d.*	*the anſwer.*

7211

7211	at	1 *s.*	3	*Anſ.* 450 *l.*	13 *s.*	9 *d.*
2701	at	3 *s.*	2	*Anſ.* 429 *l.*	1 *s.*	8 *d.*
2547	at	7 *s.*	3	*Anſ.* 923 *l.*	5 *s.*	9 *d.*
801	at	10 *s.*	9	*Anſ.* 430 *l.*	10 *s.*	9 *d.*
841	at	13 *s.*	2	*Anſ.* 553 *l.*	13 *s.*	2 *d.*
807	at	16 *s.*	5	*Anſ.* 662 *l.*	8 *s.*	3 *d.*
309	at	17 *s.*	3	*Anſ.* 266 *l.*	10 *s.*	3 *d.*
969	at	19 *s.*	11	*Anſ.* 964 *l.*	19 *s.*	3 *d.*

CASE 7.

When the price is ſhillings, pence and farthings.

RULE.

Divide the price into aliquot parts of a pound, or of one another, and the ſum of the quotients, belonging to each aliquot part, is the anſwer required.

EXAMPLES.

244 *at* 5*s.* $8\frac{1}{2}$*d.*

5 *s.* *is* $\frac{1}{4}$	61			
6 *d.* *is* $\frac{1}{10}$	6	- 2		
2 *d* *is* $\frac{1}{3}$	2	- 0	- 8	
$\frac{1}{2}$ *d.* *is* $\frac{1}{4}$	-	- 10	- 2	
	69 *l.*	- 12 *s.*	- 10 *d.*	*the anſwer.*

875	at	1 *s.*	$4\frac{3}{4}$ *d.*	*Anſ.* 61 *l.*	1 *s.*	$4\frac{1}{4}$ *d.*
7524	at	3 *s.*	$5\frac{1}{2}$ *d.*	*Anſ.* 1301 *l.*	0 *s.*	6 *d.*
3715	at	9 *s.*	$4\frac{1}{2}$ *d.*	*Anſ.* 1741 *l.*	8 *s.*	$1\frac{1}{2}$ *d.*
2572	at	13 *s.*	$7\frac{1}{2}$ *d.*	*Anſ.* 1752 *l.*	3 *s.*	6 *d.*
1603	at	16 *s.*	$10\frac{1}{2}$ *d.*	*Anſ.* 1352 *l.*	10 *s.*	$7\frac{1}{2}$ *d.*
2710	at	19 *s.*	$2\frac{1}{2}$ *d.*	*Anſ.* 2602 *l.*	14 *s.*	7 *d.*

CASE 8.

When the price is pounds, ſhillings, pence and farthings.

RULE.

Multiply the given number by the number of pounds and work for the reſt the ſhorteſt way that can be done, and theſe added together will give the anſwer.

EXAMPLES.

	428	*at* 3*l.* 4*s.* 6$\frac{1}{2}$*d.*		
	3			
	1284			
4 *s.* *is* $\frac{1}{5}$	85	- 12		
6 *d.* *is* $\frac{1}{8}$	10	- 14		
$\frac{1}{2}$ *d.* *is* $\frac{1}{12}$	—	- 17	- 10	
	1381 *l.*	- 3 *s.*	- 10 *d.*	*the anſwer.*

137 at	1 *l.* 17 *s.* 6$\frac{1}{4}$ *d.*	*Anſ.*	257 *l.* 0 *s.* 4$\frac{1}{4}$ *d.*
947 at	4 *l.* 15 *s.* 10$\frac{1}{4}$ *d.*	*Anſ.*	4538 *l.* 13 *s.* 10$\frac{3}{4}$ *d.*
457 at	14 *l.* 17 *s.* 9$\frac{1}{2}$ *d.*	*Anſ.*	6804 *l.* 10 *s.* 9$\frac{1}{2}$ *d.*
713 at	19 *l.* 19 *s.* 11$\frac{3}{4}$ *d.*	*Anſ.*	14259 *l.* 5 *s.* 1$\frac{3}{4}$ *d.*

CASE 9.

When the number whoſe price is required is a whole number, with parts annexed.

RULE.

Work for the whole number according to the former rules, to which add $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of the price, according as the queſtion requires.

EXAMPLES.

	234$\frac{3}{4}$	*at* 5*s.* 8*d.*	
5*s.* *is* $\frac{1}{4}$	58	- 10	
6*d.* *is* $\frac{1}{10}$	5	- 17	
2*d.* *is* $\frac{1}{3}$	1	- 19	
	2	- 10	*for* $\frac{1}{2}$
	1	- 5	*for* $\frac{1}{4}$
	70 *l.*	- 1 *s.*	*the anſwer.*

273

$273\frac{1}{4}$ at		2 *s.*	6 *d.*	*Anf.*	34 *l.*	3 *s.*	$1\frac{1}{2}$ *d.*
$937\frac{1}{2}$ at	3 *l.*	17 *s.*	8 *d.*	*Anf.*	3640 *l.*	12 *s.*	6 *d.*
$139\frac{3}{4}$ at	1 *l.*	19 *s.*	4 *d.*	*Anf.*	274 *l.*	16 *s.*	10 *d.*
$371\frac{3}{4}$ at	4 *l.*	13 *s.*	7 *d.*	*Anf.*	1739 *l.*	9 *s.*	$7\frac{1}{4}$ *d.*

CASE. 10.

When the quantity whofe price is required is of feveral denominations.

RULE.

Multiply the price by the number in the higheft denomination, and take the fame parts of the price for the reft as they are of an unit in the higheft number; and thefe added together, will give the anfwer.

EXAMPLES.

8 *cwt.* 2 *qr.* 16 *lb.* *at* 2 *l.* 5 *s.* 6 *d.*

	l.	*s.*	*d.*
	2	5	6
			8
	18	4	-
2 *qr.* is $\frac{1}{2}$	1	2	9
14 *lb.* is $\frac{1}{4}$	-	5	$8\frac{1}{4}$
2 *lb.* is $\frac{1}{7}$	-	-	$9\frac{3}{4}$
	19 *l.*	13 *s.*	3 *d.* *the anfwer.*

37 *cwt.* 2 *qrs.* 14 *lb.* at 7 *l.* 10 *s.* 9 *d.* *per cwt.*
Anf. 283 *l.* 11 *s.* $11\frac{1}{2}$ *d.*

17 *cwt.* 1 *qr.* 12 *lb.* at 1 *l.* 19 *s.* 8 *d.* *per cwt.*
Anf. 34 *l.* 8 *s.* 6 *d.*

23 *cwt.* 3 *qrs.* 8 *lb.* at 3 *l.* 19 *s.* 11 *d.* *per cwt.*
Anf. 95 *l.* 3 *s.* $8\frac{3}{4}$ *d.*

39 *cwt.* 0 *qr.* 10 *lb.* at 1 *l.* 17 *s.* 10 *d.* *per cwt.*
Anf. 73 *l.* 18 *s.* $10\frac{1}{4}$ *d.*

TARE AND TRETT.

Tare and *Trett* are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance made to the buyer for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at so much *per* box, &c. at so much *per cwt.* or at so much in the gross weight.

Trett is an allowance of 4 *lb.* in every 104 *lb.* for waste, dust, &c.

Cloff is an allowance of 2 *lb.* upon every 3 *cwt.*

Gross weight is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. that contains them.

Suttle is when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

CASE I.

When the tare is at so much per *box, barrel, or bag, &c.*

RULE*

Multiply the number of boxes, or barrels, &c. by the tare, and subtract the product from the gross, and the remainder is the neat weight required.

EXAMPLES.

1. In 7 frails of raisins, each weighing 5 *cwt.* 2 *qr.* 5 *lb.* gross, tare 23 *lb.* *per* frail, how much neat?

* It is manifest, that this, as well as every other case in this rule, is only an application of the rules of proportion and practice.

$23 \times 7 = 1$ *cwt.* 1 *qr.* 21 *lb.*

cwt.	*qr.*	*lb.*	
5	2	5	
		7	
38	3	7	*groſs*
1	1	21	*tare*

37 *cwt.* 1 *qr.* 14 *lb.* *the anſwer.*

2. In 241 barrels of figs, each 0 *cwt.* 3 *qr.* 19 *lb.* groſs, tare 10 *lb.* per barrel, how many pounds neat? *Anſ.* 22413 *lb.*

3. What is the neat weight of 14 *hdds.* of tobacco, each 5 *cwt.* 2 *qrs.* 17 *lb.* groſs, tare 100 *lb. per hhd?* *Anſ.* 66 *cwt.* 2 *qr.* 14 *lb.*

4. What is the neat weight of 17 bags of cotton yarn each weighing 2 *cwt.* 3 *qrs.* 4 *lb.* groſs, tare 9 *lb.* *per* bag? *Anſ.* 45 *cwt.* 3 *qr.* 27 *lb.*

CASE 2.

When the tare is at ſo much per *cwt.*

RULE.

Divide the groſs weight by the aliquot parts of a *cwt.* and ſubtract the quotient from the groſs, and the remainder is the neat weight.

EXAMPLES.

1. Groſs 173 *cwt.* 3 *qr.* 17 *lb.* tare 16 *lb. per cwt.* how much neat?

	cwt.	qr.	lb.	
	173	3	17	gross
14lb. is $\frac{1}{8}$	21	2	26	
2lb. is $\frac{1}{7}$	3	0	11	
	24	3	9	
	149	0	8	the answer.

2. What is the neat weight of 7 barrels of pot-ash, each weighing 201 *lb.* gross, tare being at 10*lb. per cwt?* *Ans.* 1281 *lb.* 6 *oz.*

3. In 25 barrels of figs, each 2 *cwt.* 1 *qr.* gross, tare 16*lb. per cwt.* how much neat? *Ans.* 48 *cwt.* 0 *qr.* 24 *lb.*

4. What is the value of the neat weight of 13 *hhds.* of tobacco, at 4 *l.* 13 *s.* 6 *d. per cwt.* each weighing 4 *cwt.* 3 *qr.* 17 *lb.* gross, tare 13 *lb. per cwt.*

Ans. 263*l.* 8*s.* 6½*d.*

CASE 3.

When Trett is allowed with Tare.

RULE.

Divide the suttle weight by 26, and the quotient is the trett, which subtract from the suttle, and the remainder is the neat.

EXAMPLES.

1. In 9 *cwt.* 2 *qr.* 17 *lb.* gross, tare 37 *lb.* and trett as usual, how much neat?

	cwt.	qr.	lb.	
	9	2	17	gross
	0	1	9	tare
26)	9	0	8	suttle
	-	1	11	trett
	8	2	25	the answer.

2. In

2. In 152 *cwt.* 1 *qr.* 3 *lb.* groſs, tare 10 *lb. per cwt.* and trett as uſual, how much neat?

Anſ. 133 *cwt.* 1 *qr.* 11 *lb.*

3. In 7 caſks of prunes, each weighing 3 *cwt.* 1 *qr.* 5 *lb.* groſs, tare $17\frac{1}{2}$ *lb. per cwt.* and trett as uſual, how much neat? *Anſ.* 18 *cwt.* 2 *qr.* 25 *lb.*

4. What is the neat weight of 3 *hhds.* of ſugar, weighing as follows: the 1ſt. 4 *cwt.* 0 *qr.* 5 *lb.* groſs, tare 73 *lb*; the 2d. 3 *cwt.* 2 *qr.* groſs, tare 56 *lb*; and the 3d. 2 *cwt.* 3 *qr.* 17 *lb.* groſs, tare 47 *lb.* and allowing trett to each as uſual? *Anſ.* 8 *cwt.* 2 *qr.* 4 *lb.*

CASE 4.

When tare, trett, and cloff are all allowed.

RULE.

Deduct the tare and trett, as before, and divide the ſuttle by 168, and the quotient is the cloff, which ſubtract from the ſuttle, and the remainder is the neat.

EXAMPLES.

What is the neat weight of a *hhd.* of tobacco, weighing 15 *cwt.* 3 *qr.* 20 *lb.* groſs, tare 7 *lb. per cwt.* and trett and cloff as uſual.

	cwt.	qr.	lb.	
	15	3	20	*groſs*
7 *lb.* is $\frac{1}{16}$	—	3	27	*tare*
26)	14	3	21	
	—	2	8	*trett*
168)	14	1	13	*ſuttle*
	—	-	9	*cloff*
	14	1	4	*the anſwer.*

 2. In

2. In 19 chests of sugar, each containing 13 *cwt.* 1 *qr.* 17 *lb.* gross, tare 13 *lb. per cwt.* and trett and cloff as usual, how much neat, and what is the value at $5\frac{3}{4}$*d. per lb?*

Ans. 215 *cwt.* 0 *qr.* 17 *lb. and value* 577*l.* 6*s.* $5\frac{3}{4}$*d.*

3. 29 parcels, each weighing 3*cwt.* 0*qr.* 14*lb.* gross; what is the value of the neat weight at 1 *l.* 11 *s.* 6 *d. per cwt.* allowing 8 *lb. per cwt.* for tare, and trett and cloff as usual? *Ans.* 126*l.* 14*s.* $0\frac{3}{4}$*d.*

BILLS OF PARCELS.

A Hosier's Bill.

Mr. Thomas Williams

Bought of Richard Simpson, Jan. 4, 1776.

			s.	d.	
8	Pair of worsted stockings,	at	4	6	*per pair.*
5	Pair of thread ditto,	at	3	2	
3	Pair of black silk ditto,	at	14	0	
6	Pair of black worsted ditto,	at	4	2	
4	Pair of cotton ditto,	at	7	6	
2	Yards of fine flannel ditto	at	1	8	*per yard.*
					£. 7 12 2

A Mercer's Bill.

Mr. William George

Bought of Peter Thompson, July 13, 1776.

			s.	d.	
15	Yards of sattin,	at	9	6	*per yard.*
18	Yards of flowered silk	at	17	4	
12	Yards of rich brocade,	at	19	8	
16	Yards of sarsnet,	at	3	2	
13	Yards of Genoa velvet,	at	27	6	
23	Yards of lutestring,	at	6	3	
					£. 62 2 5

A Linen-

A Linen-Draper's Bill.

Mr. Henry Morris

Bought of Caleb Windsor, March 8, 1776.

			s.	d.	
40	Ells of dowlas,	at	1	6	*per ell.*
34	Ells of diaper,	at	1	4½	
31	Ells of holland,	at	5	8	
39	Yards of Irish cloth,	at	2	4	*per yard.*
17½	Yards of muslin,	at	7	2½	
13¾	Yards of cambric,	at	10	6	
27	Yards of printed linen,	at	2	5	

35 9 2¼

A Milliner's Bill.

Mrs. Matthewson

Bought of Simon Percy, June 18, 1776.

			l.	s.	d.	
18	Yards of fine lace,	at	0	12	3	*per yard.*
5	Pair of fine kid gloves,	at	0	2	2	*per pair.*
12	Fans with French mounts,	at	0	3	6	*each.*
2	Fine laced tippets,	at	3	3	0	
4	Dozen of linen gloves,	at	0	1	3	*per pair.*
6	Sets of knots,	at	0	2	6	*per set.*

£. 23 14 4

A Woollen-Draper's Bill.

Mr. John Page

Bought of Jacob Goodson, May 1, 1776.

			l.	s.	d.	
17	Yards of fine serge,	at	0	3	9	*per yard.*
18	Yards of drugget,	at	0	9	0	
15	Yards of superfine scarlet	at	1	2	0	
16	Yards of super. black cloth,	at	0	18	0	
25	Yards of shalloon,	at	0	1	9	
17	Yards of drab,	at	0	17	6	

£. 59 5 0

A Grocer's Bill.

Mr. Nathaniel Parsons

Bought of William Smith, Aug. 6, 1776.

		s.	d.	
$24\frac{1}{2}$ *lb.* of royal green tea,	at	18	6	*per lb.*
$24\frac{1}{4}$ *lb.* of imperial tea,	at	24	0	
$35\frac{3}{4}$ *lb.* of best bohea,	at	13	10	
17 *lb.* of coffee,	at	5	4	
25 *lb.* of double refined sugar,	at	1	$1\frac{1}{2}$	
9 Sugar loaves, wt. 137 *lb.*	at	0	$7\frac{1}{2}$	

£. 83 2 $2\frac{1}{2}$

A Wine Merchant's Bill.

Mr. Thomas Greville

Bought of John Simes, April 3, 1776.

		s.	d.	
12 Gallons of palm sack,	at	8	6	*per gall.*
17 Gallons of red port,	at	5	8	
9 Gallons of claret,	at	8	9	
34 Gallons of white lisbon,	at	4	10	
$22\frac{1}{2}$ Gallons of rhenish,	at	6	4	
$27\frac{3}{4}$ Gallons of sherry,	at	6	2	

£. 37 15 $0\frac{1}{2}$

A Cheesemonger's Bill.

Mr. Edward Patterson

Bought of Stephen Cross, Sept. 1, 1776.

		s.	d.	
8 *lb.* of Cambridge butter,	at	0	6	*per lb.*
17 *lb.* of new cheese,	at	0	4	
$\frac{1}{2}$ Firkin of butter, wt. 28*lb.*	at	0	$5\frac{1}{2}$	
5 Cheshire cheeses, wt. 127*lb.*	at	0	4	
2 Warwickshire ditto, wt. 15*lb.*	at	0	3	
2 *lb.* of cream cheese,	at	0	6	

£. 3 14 7

SIMPLE INTEREST.

Simple Interest is a gratuity allowed by the borrower of any ſum of money to the lender, according to a certain rate *per cent.* agreed on; which, by law, muſt not exceed 5*l.* that is, 5*l.* for the uſe of 100*l.* 1 year; 10*l.* for the uſe of it 2 years; and ſo on.

Principal is the money lent.

Rate is the ſum *per cent.* agreed on.

Amount is the principal and intereſt added together.

RULE*.

1. Multiply the principal by the rate, and divide the product by 100, and the quotient is the anſwer for 1 year.

2. Multiply the intereſt for 1 year by the time given, and the product is the anſwer for that time.

3. If there be parts of a year, as months or days, work for the months by the aliquot parts of a year, and for the days by the rule of three direct.

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* There are ſome caſes where it is cuſtomary to conſider the time elapſed different ways. In the courts of law, intereſt is always computed in years, quarters and days; which, indeed, is the only equitable method: but in computing the intereſt on the public bonds of the South Sea and India companies, and in the Bank of England, &c. the time is generally taken in calendar months and days; and on Exchequer bills in quarters of a year and days.

EXAM-

EXAMPLES.

1. What is the intereſt of 284 *l.* 10 *s.* for 2 years, 4 months, and 25 days, at $3\frac{1}{2}$ *per cent. per annum.*

284*l.* 10*s.*	365 : 9*l.* 19*s.* $1\frac{3}{4}$*d* : : 25 *days*
$3\frac{1}{2}$	5
853 10	49 15 $8\frac{3}{4}$
142 5	5
9.95 15	365)248 18 $7\frac{3}{4}$(13*s.* $7\frac{1}{2}$*d.*
20	20
19.15	4978
12	1328
1.80	233
4	12
3.20	2803
	248
	4
	995
	265

9*l.* 19*s.* $1\frac{3}{4}$*d.* = 1 *year's intereſt.*
2

	19	18	$3\frac{1}{2}$	= 2 *year's intereſt.*
4 *mo.* = $\frac{1}{3}$	3	6	$4\frac{1}{2}$	= 4 *month's ditto*
		13	$7\frac{1}{2}$	= 25 *days ditto*
	23	18	$3\frac{1}{2}$	*the anſwer required.*

2. What is the intereſt of 230 *l.* 10s. for 1 year at 4 *per cent. per annum?* *Anſ.* 9*l.* 4*s.* $4\frac{3}{4}$*d.*

4. What

3. What is the intereſt of 547*l.* 15*s.* for 3 years, at 5 *per cent. per annum*? *Anſ.* 82*l* 3*s.* 3*d.*

4. What is the amount of 690*l.* for three years, at $4\frac{1}{4}$ *per cent. per. ann?* *Anſ.* 777*l.* 19*s.* 6*d.*

5. What is the intereſt of 205*l.* 15*s.* for $\frac{1}{4}$ year at 4 *per cent. per ann?* *Anſ.* 2*l.* 1*s.* $1\frac{3}{4}$*d.*

6. What is the amount of 120*l.* 10*s.* for $2\frac{1}{2}$ years, at $4\frac{3}{4}$ *per cent. per ann?* *Anſ.* 134*l.* 16*s.* $1\frac{3}{4}$*d.*

7. What is the intereſt of 47 *l.* 10 *s.* for $4\frac{3}{4}$ years, and 52 days, at $4\frac{1}{2}$ *per cent?* *Anſ.* 10 *l.* 9 *s.* $1\frac{3}{4}$*d.*

8. What is the amount of 200 guineas for 4 years, 7 months and 25 days, at $4\frac{1}{2}$ *per cent?*.

Anſ. 253 *l.* 19 *s.* $2\frac{1}{4}$ *d.*

9. A gentleman left his niece by will 558*l.* 15*s.* to be paid her when ſhe came to age, with intereſt at 4 *per cent.* now ſhe came to age in 5 years, 9 months and 21 days; what has ſhe to receive in all?

Anſ. 688 *l.* 10*s.* $11\frac{1}{2}$*d.*

10. What is the intereſt due upon an Indian bond of 500*l.* value, at $3\frac{1}{2}$ *per cent. per ann.* from September 30, 1763, to June 18, 1764? *Anſ.* 12 *l.* 10 *s.* $3\frac{1}{4}$*d.*

11. What is the intereſt due upon an Exchequer bill of 450*l.* at $3\frac{1}{4}$ *per cent. per ann.* for $2\frac{3}{4}$ *years* and 67 *days?* *Anſ.* 49 *l.* 10 *s.* $0\frac{3}{4}$ *d.*

COMMISSION.*

Commiſſion is an allowance of ſo much *per cent.* to a factor or correſpondent abroad for buying and ſelling goods for his employer.

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* The method of working queſtions in this and the following rules of inſurance, brokerage &c. is the ſame as in ſimple intereſt.

EXAMPLES.

1. What comes the commission of 500*l.* 13*s.* 6*d.* to at $3\frac{1}{2}$ *per cent?*

£.	s.	d.
500 -	13 -	6
		$3\frac{1}{2}$
1502 -	0 -	6
250 -	6 -	9
17.52 -	7 -	3
20		
10.47		
12		
5.67		
4		
2.68		

Ans. 17 *l.* 10*s.* $5\frac{1}{2}$*d.*

2. My correspondent writes me word that he has bought goods on my account to the value of 754*l.* 16*s.* what does his commission come to at $2\frac{1}{2}$ *per cent?*

Ans. 18*l.* 17*s.* $4\frac{3}{4}$*d.*

3. What must I allow, my correspondent for disbursing on my account 529*l.* 18*s.* 5*d.* at $2\frac{1}{4}$ *per cent?*

Ans. 11 *l.* 18 *s.* 5 $\frac{1}{2}$*d.*

4. If I allow my factor 7 $\frac{5}{8}$ *per cent.* for commission, what may he demand on the laying out 1200*l.*?

Ans. 91*l.* 10*s.*

5. What does the commission on 950*l.* come to at $3\frac{7}{8}$ *per cent?* *Ans.* 36*l.* 16*s.* 3*d.*

BROKERAGE.

Brokerage is an allowance of ſo much *per cent.* to a perſon called a broker, for aſſiſting merchants or factors in procuring or diſpoſing of goods.

Examples.

1. What is the brokerage of 610*l.* at 5*s.* or $\frac{1}{4}$ *per cent?*

```
             £.
5 s. is 1/4  610
            ------
            1.52 - 10
              20
            ------
            10.50
              12
            ------
             6.00
```

Anſ. 1 *l.* 10 *s.* 6 *d.*

2. If I allow my broker $3\frac{3}{4}$ *per cent.* what may he demand when he ſells goods to the value of 876 *l.* 5 *s.* 10 *d.* *Anſ.* 32 *l.* 17 *s.* $2\frac{1}{2}$ *d.*

3. What is the brokerage of 879 *l.* 18*s.* at $\frac{3}{8}$ *per cent?* *Anſ.* 3 *l.* 5 *s.* $11\frac{3}{4}$ *d.*

4. If a broker ſells goods to the amount of 508 *l.* 17 *s.* 10 *d.* what is his demand at $1\frac{1}{2}$ *per cent.*

Anſ. 7 *l.* 12 *s.* 8 *d.*

INSURANCE.

Inſurance is a premium of ſo much *per cent.* given to certain perſons and offices for a ſecurity of making good the loſs of ſhips, houſes, merchandizes, &c. which may happen from ſtorms, fire, &c.

Ex-

EXAMPLES.

1. What is the insurance of 874*l.* 13*s.* 6*d.* at $13\frac{1}{2}$ *per cent?*

£.	s.	d.
874	13	6
		12
10496	2	0
874	13	6
437	6	9
118.08	2	3
20		
1.62		
12		
7.47		
4		
1.88		

Ans. 118*l.* 1*s.* $7\frac{1}{2}$*d.*

2. What is the insurance of 900 *l.* at $10\frac{3}{4}$ *per cent?*
Ans. 96 *l.* 15 *s.*

3. What is the insurance of 1200 *l.* at $7\frac{5}{8}$ *per cent?*
Ans. 91 *l.* 10 *s.*

4. What is the insurance of an East-India ship and cargo valued at 35727 *l.* 17 *s.* 6 *d.* at $17\frac{7}{8}$ *per cent.?*
Ans. 6386*l.* 7*s.* $1\frac{3}{4}$*d.*

BUYING AND SELLING OF STOCKS.

Stock is a general name for the capitals of our trading companies, and the buying and selling certain sums of money in those funds is now become a general practice.

EXAMPLES.

1. What is the purchase of 2054*l.* 16*s.* South Sea stock, at $110\frac{1}{4}$ *per cent?*

	£.	s.	
10 is $\frac{1}{10}$	2054	- 16	
	205	- 9	- 7
$\frac{1}{4}$ is $\frac{1}{40}$	5	- 2	- $8\frac{3}{4}$
	2265*l.*	- 8*s.*	- $3\frac{3}{4}$ *the answer.*

2. What is the purchase of 156*l.* 15*s.* 3 *per cent.* annuities, at $74\frac{1}{2}$ *per cent.?* *Ans.* 116*l.* 15*s.* $6\frac{3}{4}$*d.*

3. What is the purchase of 816*l.* 12*s.* bank annuities, at $89\frac{3}{8}$ *per cent?* *Ans.* 729*l.* 16*s.* $8\frac{1}{2}$*d.*

4. What is the purchase of 987*l.* 15*s.* India stock, at $113\frac{7}{8}$ *per cent?* *Ans.* 1124*l.* 16*s.*

5. Bought 650*l.* bank annuities at $90\frac{3}{8}$ *per cent.* and paid brokerage $\frac{1}{8}$ *per cent.* what did the whole amount to? *Ans.* 588*l.* 5*s.*

6. What does 2400*l.* capital stock in the 3 *per cent.* consolidated bank annuities come to, at $84\frac{1}{8}$ *per cent.?* *Ans.* 2019*l.*

DISCOUNT.

Discount is an allowance made for the payment of any sum of money before it becomes due; and is the difference between that sum due some time hence, and its present worth.

The *present worth* of any sum, or debt, due some time hence, is such a sum as, if put to interest, would, in that time, and at the rate *per cent.* for which the discount is to be made, amount to the sum or debt then due.

RULE*.

1. As the amount of 100*l.* for the given rate and time is to 100*l.*

So

* That an allowance ought to be made for paying money before it becomes due, which is supposed to bear no interest till after it is due

So is the given ſum or debt to the preſent worth.

2. Subtract the preſent worth from the given ſum, and the remainder is the diſcount required.

---

due, is very reaſonable; for if I keep the money in my own hands till the debt becomes due, it is plain I may make an advantage of it by putting it out to intereſt for that time; but if I pay it before it is due, it is giving that benefit to another; therefore we have only to enquire what diſcount ought to be allowed. And here ſome debtors will be ready to ſay, that ſince by not paying the money till it becomes due, they may employ it at intereſt, therefore by paying it before due, they ſhall loſe that intereſt, and, for that reaſon, all ſuch intereſt ought to be diſcounted: but that is falſe, for they cannot be ſaid to loſe that intereſt till the time the debt becomes due arrives; whereas we are to conſider what would properly be loſt at preſent, by paying the debt before it becomes due; and this can, in point of equity or juſtice, be no other than ſuch a ſum, as being put out to intereſt till the debt becomes due, would amount to the intereſt of the debt for the ſame time.—It is, beſides, plain, that the advantage ariſing from diſcharging a debt, due ſome time hence, by a preſent payment, according to the principles we have mentioned, is exactly the ſame as employing the whole ſum at intereſt till the time the debt becomes due arrives: for if the diſcount allowed for preſent payment be put out to intereſt for that time, its amount will be the ſame as the intereſt of the whole debt for the ſame time: thus, the diſcount of 105*l.* due one year hence, reckoning intereſt at 5 *per cent.* will be 5*l.* and 5*l.* put out to intereſt at 5 *per cent.* for one year will amount to 5*l.* 5*s.* which is exactly equal to the intereſt of 105*l.* for one year at 5 *per cent.*

The truth of the rule for working is evident from the nature of ſimple intereſt: for ſince the debt may be conſidered as the amount of ſome principal (called here, the preſent worth) at a certain rate *per cent.* and for the given time, that amount muſt be in the ſame proportion, either to its principal or intereſt, as the amount of any other ſum, at the ſame rate, and for the ſame time, is to its principal or intereſt.

The method uſed amongſt Bankers, &c. in diſcounting bills, is to find the intereſt of the ſum drawn for, from the time the bill is diſcounted to the time when it becomes due, (including the days of grace) which intereſt they reckon as the diſcount, thereby making it more than it really is.

But when goods are bought or ſold, and diſcount is to be made for preſent payment, at any rate *per cent.* without regard to time, the intereſt of the ſum as calculated for a year is the diſcount.

2. Subtract

Or,

As the amount of 100 *l.* for the given rate and time is to the intereſt of 100 *l.* for that time,

So is the given ſum or debt to the diſcount required.

EXAMPLES.

1. What is the diſcount of 573 *l.* 15 *s.* due 3 years hence, at $4\frac{1}{2}$ *per cent*?

```
 £.   s.
 4 - 10
      3
 -------
13 - 10
100
 -------          £.   s.      £.   s.
113 - 10    :    13 - 10  ::  573 - 15
 20               20            20
 ----             ----         -----
2270              270          11475
                                 270
                              -------
                              803250
                              22950
                             --------   (20)
                       227,0)309825,0(136,4
                              828       -------
                              1472      68 - 4
                              1105
                               197
                                12
                              ----
                         227)2364(10
                               94
                                4
                              ----
                              376(1/4
                              149
```

Anſ. 68 *l.* 4 *s.* $10\frac{1}{4}$ *d.*

2. What is the present worth of 150*l.* payable in $\frac{3}{4}$ year, discounting at five *per cent?* *Ans.* 148*l.* 2*s.* 11$\frac{1}{2}$*d.*

3. What is the present worth of 75*l.* due 15 months hence, at 5 *per cent?* *Ans.* 70*l.* 11*s.* 9*d.*

4. What is the discount on 85 *l.* 10 *s.* due September 8, this being July 4, reckoning interest at 5 *per cent. per annum?* *Ans.* 15 *s.* 3$\frac{1}{2}$ *d.*

5. What ready money will discharge a debt of 543 *l.* 7 *s.* due 4 months and 18 days hence at 4$\frac{5}{8}$ *per cent. per annum?* *Ans.* 533 *l.* 18 *s.* 0$\frac{1}{4}$ *d.*

6. Bought a quantity of goods for 150 *l.* ready money, and sold them again for 200 *l.* payable at $\frac{3}{4}$ of a year hence; what was the gain in ready money, supposing discount to be made at 5 *per cent?*

Ans. 42 *l.* 15 *s.* 5 *d.*

7. What is the present worth of 120 *l.* payable as follows; viz. 50*l.* at 3 months; 50 *l.* at 5 months, and the rest at 8 months, discounting at 6 *per cent?*

Ans. 117 *l.* 5 *s.* 5$\frac{1}{4}$ *d.*

COMPOUND INTEREST.

Compound Interest is that which arises from the principal and interest taken together, as it becomes due, at the end of each stated time of payment.

RULE.*

1. Find the amount of the given principal, for the time of the first payment, by simple interest.

2. Consider this amount as the principal for the second payment, whose amount calculate as before, and so on through all the payments to the last, still accounting the last amount as the principal for the next payment.

EXAMPLES.

1. What is the amount of 320*l.* 10*s.* for four years, at 5 *per cent. per annum*, compound interest.

$\frac{1}{20}$

$\frac{1}{20}$)	320*l.*	10*s.*		*1st. year's principal*
	16	—	6	*1st. year's interest.*
$\frac{1}{20}$)	336	10	6	*2d. year's principal*
	16	16	$6\frac{1}{4}$	*2d. year's interest.*
$\frac{1}{20}$)	353	7	-$\frac{1}{4}$	*3d. year's principal*
	17	13	4	*3d. year's interest.*
$\frac{1}{20}$)	371	—	$4\frac{1}{4}$	*4th. year's principal*
	18	11	-	*4th. year's interest.*
	389	11	$4\frac{1}{4}$	*whole amount, or the answer required.*

2. What is the compound interest of 760*l.* 10*s.* forborn 4 years at 4 *per cent ?* *Ans.* 129*l.* 3*s.* $6\frac{1}{4}$*d.*

What is the amount of 15*l.* 10*s.* for 9 years, at $3\frac{1}{2}$ *per cent. per annum*, compound interest ?

Ans. 21*l.* 2*s.* $4\frac{1}{4}$*d.*

4. What is the compound interest of 410*l.* forborn for $2\frac{1}{2}$ years at $4\frac{1}{2}$ *per cent. per annum* ; the interest payable half yearly ? *Ans.* 48*l.* 4*s.* $11\frac{3}{4}$*d.*

5. Find the several amounts of 50*l.* payable yearly, half yearly and quarterly, being forborn 5 years, at 5 *per cent. per annum*, compound interest ?

Ans. 63*l.* 16*s.* 3*d*$\frac{1}{4}$, 64*l.* 0*s.* 0*d.* and 64*l.* 1*s.* $9\frac{1}{2}$*d.*

EQUATION OF PAYMENTS.

Equation of Payments is the finding a time, to pay at once, several debts due at different times, so that no loss shall be sustained by either party.

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* The reason of this rule is evident from the definition, and the principles of simple interest.

RULE*.

Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the time required.

EXAMPLES.

A owes B 190*l.* to be paid as follows, viz. 50*l.* in 6 months, 60*l.* in 7 months, and 80*l.* in 10 months; what is the equated time to pay the whole?

$$50 \times 6 = 300$$
$$60 \times 7 = 420$$
$$80 \times 10 = 800$$

$$50 + 60 + 80 = 190)\ 1520\ (8$$
$$1520$$

Answer 8 *months*.

2. A owes B. 52*l.* 7*s.* 6*d.* to be paid in $4\frac{1}{2}$ months, 80*l.* 10*s.* to be paid in $3\frac{1}{2}$ months, and 76*l.* 2*s.* 6*d.* to be paid in 5 months; what is the equated time to pay the whole? *Ans.* 4 *mo.* 8 *da.*

3. A

* This rule is founded upon a supposition, that the sum of the interests of the several debts which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Among others, that defend this principle, Mr. Cocker endeavours to prove it to be right by this argument: that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due: but this cannot be the case; for though by keeping a debt unpaid after it is due there is gained the interest of it for that time, yet by paying a debt before it is due, the payer does not lose the interest for that time, but the dis-

3. A owes B 240*l.* to be paid in 6 months, but in 1 month and a half, pays him 60 *l.* and in $4\frac{1}{2}$ months after that 80 *l.* more: how much longer than 6 months should B in equity defer the rest? *Ans.* $3\frac{9}{15}$ *months.*

4. A debt is to be paid as follows: viz. $\frac{1}{4}$ at 2 months, $\frac{1}{8}$ at 3 months, $\frac{1}{8}$ at 4 months, $\frac{1}{8}$ at 5 months, and the rest at 7 months: what is the equated time to pay the whole? *Ans.* 4 *months* & 18 *days.*

BARTER.

Barter is the exchanging of one commodity for another, and directs traders so to proportion their goods, that neither party may sustain loss.

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discount only, which is less than the interest, and therefore the rule is not true.

Although this rule be not accurately true, yet in most questions that occur in business, the error is so trifling that it will always be made use of as the most eligible method.

That the rule is universally agreeable to the supposition may be thus demonstrated.

Let $\begin{cases} d = \text{first debt payable, and the distance of its term of payment } t. \\ D = \text{last debt payable, and the distance of its term } T. \\ x = \text{distance of the equated time.} \\ r = \text{rate of interest of 1 } l. \text{ for one year.} \end{cases}$

Then, since x lies between T and t $\begin{cases} \text{The distance of the time } t \text{ and } x \text{ is } = x - t. \\ \text{The distance of the time T and } x \text{ is } = T - x. \end{cases}$

Now the interest of d for the time $x - t$ is $\overline{x-t} \times dr$; and the interest of D for the time T—x is $\overline{T-x} \times Dr$; therefore $\overline{x-t} \times dr = \overline{T-x} \times Dr$ by the supposition; and from this equation x is found $= \dfrac{DT + dt}{D + d}$, which is the rule. And the same might be shewn of any number of payments.

The true rule will be given in equation of payments by decimals.

RULE*

Find the value of that commodity whose quantity is given; then find what quantity of the other, at the rate proposed, you may have for the same money, and it gives the answer required.

EXAMPLES.

1. How many dozen of candles, at 5*s.* 2*d.* *per doz.* must be given in barter for 3 *cwt.* 2 *qrs.* of tallow at 37 *s.* 4*d.* *per cwt*?

qr.		s.	d.		cwt.	qr.
4	:	37	4	::	3	2
		12			4	
		448			14	
		14				
		1792				
		448				
		4)6272				
		12)1568				
		2,0)13,0—8				
		6*l.* 10*s.* 8*d.*				

* This rule is, evidently, only an application of the rule of three direct.

s.	d.		doz.		l.	s.	d.
5	2	:	1	::	6	10	8
12					20		
62					130		
					12		

62)1568(25
124

328
310

18
12

62)216(3
186

Ans. 25 *doz.* 3 *lb.* 30

2. How much sugar, at 8*d. per lb.* must be given in barter for 20 *cwt.* of tobacco, at 3*l. per cwt?*

Ans. 16 *cwt.* 0 *qrs.* 8 *lb.*

3. How much tea at 9*s per lb.* can I have in barter, for 4 *cwt.* 2 *qrs.* of chocolate at 4*s. per lb?* *Ans.* 2 *cwt.*

4. How many reams of paper, at 2*s.* $9\frac{1}{2}$*d. per* ream must be given in barter for 37 pieces of Irish cloth, at 1*l.* 12*s.* 4*d. per piece?* *Ans.* $428\frac{36}{67}$.

5. A merchant hath 1000 yards of canvass at $9\frac{1}{2}$ *per yard*, which he barters for serge at $10\frac{1}{4}$*d. per yard*, how many yards must he receive? *Ans.* $926\frac{34}{41}$.

6. A delivered 3 *hhds.* of brandy, at 6*s.* 8*d. per gall.* to B, for 126 yards of cloth; what was the cloth *per yard?* *Ans.* 10*s.*

 8. A

7. A and B barter, A hath 41 *cwt.* of hops, at 30*s.* *per cwt.* for which B gives him 20 *l.* in money, and the rest in prunes at 5*d.* *per lb.* what quantity of prunes must A receive? *Ans.* 17 *cwt.* 3 *qrs.* 4 *lb.*

8. A has a quantity of pepper, wt. neat 1600 *lb.* at 17 *d.* *per lb.* which he barters with B for two sorts of goods, the one at 5*d.* the other at 8*d.* *per lb.* and to have $\frac{1}{3}$ in money, and of each sort of goods an equal quantity: how many *lb.* of each must he receive, and how much in money. *Ans.* $1394\frac{38}{39}$ *lb.* and 37 *l.* 15*s.* $6\frac{2}{3}$*d.*

LOSS AND GAIN.

Loss and Gain is a rule that discovers what is got or lost in the buying or selling of goods; and instructs merchants and traders to raise or fall the price of their goods, so as to gain or lose so much *per cent*, &c.

Questions in this rule are performed by the rule of three direct.

EXAMPLES.

1. How must I sell tea *per lb.* that cost me 13*s.* 5*d.* to gain after the rate of 25 *per cent*?

£.		£.		s.	d.
100	:	125	: :	13	5

```
                     12
                    ----
                    161
                    125
                    ----
                    805
                   322
                  161
                  -----
            1,00)201,25
                 -------
             12)201—25
                -------
           16s. 9d.—1 25/100 the answer
```

Or—thus,

$$\begin{array}{l} 4)\,13\,s.\ 5\,d. \\ \quad\ 3\quad 4 \\ \hline 16\,s.\ 9\tfrac{1}{4}\,d. \end{array}$$ *the same as before.*

2. At $1\frac{1}{2}d.$ in the shilling profit, how much *per cent?* *Ans.* 12 *l.* 10 *s.*

3. At 3 *s.* 6 *d.* in the pound profit, how much *per cent?* *Ans.* 17 *l.* 10 *s.*

4. If a *lb.* of tobacco cost 16*d.* and is sold for 20*d.* what is the gain *per cent?* *Ans* 25*l*.

5. Bought goods at $4\frac{1}{2}d.$ *per lb.* and sold them at the rate of 2*l.* 7*s.* 4*d.* *per cwt.* what was the gain *per cent?* *Ans.* 12 *l.* 13*s.* 11 *d.*

6. Bought cloth at 7*s.* 6*d.* *per yard*, which not proving so good as I expected I am resoved to lose $17\frac{1}{2}$ *per cent.* by it: how must I sell it *per yard?* *Ans.* 6 *s.* $2\frac{1}{4}$ *d.*

7. Bought goods at 2 guineas *per cwt.* and sold them again retail at $5\frac{1}{4}d.$ *per lb.* what was the gain *per cent?* *Ans.* 16 *l.* 13 *s.* 4 *d.*

8. If I buy $17\frac{1}{2}$ *cwt* of sugar for 35 guineas, and retail it at $7\frac{1}{2}$ *d.* *per lb.* what shall I gain *per cent?* *Ans.* 66 *l.* 13 *s.* 4 *d.*

9. If I buy tobacco at 10 guineas *per cwt.* at what rate must I retail it *per lb.* to gain twelve *per cent?* *Ans.* 2 *s.* 1 $d.\frac{19}{112}$.

10. If, when I sell cloth at 7*s.* *per yard*, I gain 10 *per cent.* what will be the gain *per cent.* when it is sold for 8*s.* 6*d.* *per yard?* *Ans.* 33*l.* 11*s.* $5\frac{1}{11}d.$

11. If I buy 28 pieces of stuffs at 4*l.* *per piece*, and sell 13 of the pieces at 6*l.* and 8 at 5*l.* *per piece*: at what rate *per piece* must I sell the rest to gain 20 *per cent.* by the whole? *Ans.* 2 *l.* 6 *s.* $10\frac{1}{4}$ *d.*

12. Bought 40 gallons of brandy at 3*s.* *per gall.* but by accident 6 gallons of it are loſt, at what rate muſt I ſell the remainder *per* gallon, and gain upon the whole prime coſt, at the rate of 10 *per cent?*

Anſ. 3 *s.* 10$\frac{1}{2}$ *d.*

13. Bought hoſe in London at 4 *s.* 3 *d.* *per pair*, and ſold them afterwards in Dublin at 6 *s.* the pair; now taking the charge at an average to be 2*d.* the pair, and conſidering that I muſt loſe 12 *per cent.* by remitting my money home again; what do I gain *per cent.* by this article of trade? *Anſ.* 19 *l.* 10 *s.* 11 *d.*

14. Sold a repeating watch for 50 guineas, and by ſo doing loſt 17 *per cent.* whereas I ought in dealing to have cleared 20 *per cent.* how much was it ſold for under the juſt value? *Anſ.* 23 *l.* 8 *s.* ~~0$\frac{3}{4}$ *d.*~~

FELLOWSHIP.

Fellowſhip is a general rule, by which merchants, &c. trading in company, with a joint ſtock, determine each perſon's particular ſhare of the gain or loſs in proportion to his ſhare in the joint ſtock.

By this rule a bankrupt's eſtate may be divided amongſt his creditors, as alſo legacies adjuſted, when there is a deficiency of aſſets or effects.

SINGLE FELLOWSHIP.

Single Fellowſhip is when different ſtocks are employed for any certain equal time.

RULE*.

As the whole ſtock is to the whole gain or loſs,

So is each man's particular ſtock, to his particular ſhare of the gain or loſs.

---

* That the gain or loſs in this rule, is in proportion to their ſtocks is evident: for, as the times the ſtocks are in trade are equal, if I put in $\frac{1}{2}$ of the whole ſtock, I ought to

Method of PROOF.

Add all the ſhares together, and the ſum will be equal to the gain or loſs, when the queſtion is right.

EXAMPLES.

1. Two perſons trade together, A put into ſtock 130*l.* and B 220*l.* and they gained 500*l.* what is each perſon's ſhare thereof?

$$130 + 220 = 350$$

$$350 : 500 :: 130$$

```
          500
          130
        -----
        15000
          500
        ------
35,0)6500,0(185 l.
      300
       200
        25
        20
      -----
   35)500(14 s.
      150
       10
       12
      ----
   35)120(3 d.
       15
        4
      ----
   35)60(1/4
       25
```

---

to have ½ of the whole gain; if my part of the whole ſtock be $\frac{1}{3}$, my ſhare of the whole gain or loſs ought to be $\frac{1}{3}$ alſo. And, generally, if I put in $\frac{1}{n}$ of the ſtock, I ought to have $\frac{1}{n}$ part of the whole gain or loſs; that is, the ſame ratio that the whole ſtock has to the whole gain or loſs, muſt each perſon's particular ſtock have to his reſpective gain or

350 : 500 :: 220
220

10000
1000

3,50)11000,0(314 *l.*
50
150
10
20

35)200(5 *s.*
25
12

35)300(8 *d.*
20
4

35)80($\frac{1}{2}$
10

185 *l.*	14 *s.*	$3\frac{1}{4}$ *d.* $\frac{25}{35}$	= *A's share.*
314 *l.*	5 *s.*	$8\frac{1}{2}$ *d.* $\frac{10}{35}$	= *B's share.*
500 *l.*	4 *s.*	0 *d.*	*the Proof.*

2. A and B have gained by trading 182 *l.* A put into stock 300*l.* and B 400*l.* what is each person's share of the profit? *Ans.* A 78*l.* *and* B 104*l.*

3. Divide 120*l.* between three persons, so that their shares shall be to each other as 1, 2 and 3 respectively. *Ans.* 20*l.* 40*l.* *and* 60*l.*

4. Three

4. Three persons make a joint stock; A put in 184*l.* 10*s.* B 96*l.* 15*s.* and C 76*l.* 5*s.* they trade and gain 220*l.* 12*s.* what is each person's share of the gain?

Ans. A 113*l.* 16*s.* $\frac{688}{715}$, B 59*l.* 14*s.* $\frac{12}{715}$, C 47*l.* 1*s.* $\frac{15}{715}$.

5. Four persons in partnership, A, B, C and D put into stock 180*l.* 240*l.* 350*l.* and 430*l.* respectively, for 5 years certain, and at the end of that time they find they have gained 3600*l.* what is each person's share of the gain?

Ans. A 540*l.* B 720*l.* C 1050*l.* *and* D 1290*l.*

6. Three merchants, A, B and C freight a ship with 340 tuns of wine; A loaded 110 tun, B 97, and C the rest. In a storm the seamen were obliged to throw 85 tuns overboard; how much must each sustain of the loss? *Ans.* A $27\frac{1}{2}$, B $24\frac{1}{4}$, and C $33\frac{1}{4}$.

7. A ship worth 860*l.* being entirely lost, of which $\frac{1}{8}$ belonged to A, $\frac{1}{4}$ to B, and the rest to C; what loss will each sustain, supposing 500*l.* of her to be insured?

Ans. A 45*l.* B 90*l.* *and* C 225*l.*

8. A bankrupt is indebted to A 275*l.* 14*s.* to B 304*l.* 7*s.* to C 152*l.* and to D 104*l.* 6*s.* His estate is worth only 675*l.* 15*s.* how must it be divided?

Ans. A 222*l.* 15*s.* 2*d.* B 245*l.* 18*s.* $1\frac{1}{2}$*d.* C 122*l.* 16*s.* $2\frac{3}{4}$*d.* *and* D 84*l.* 5*s.* 5*d.*

9. A and B venturing equal sums of money, clear by joint trade 154*l.* By agreement A was to have 8 *per cent.* because he spent his time in the execution of the project, and B was to have only 5 *per cent*; what was A allowed for his trouble? *Ans.* 35*l.* 10*s.* $9\frac{3}{13}$.

DOUBLE FELLOWSHIP.

Double Fellowship is when different or equal stocks are employed for different times.

RULE*

Multiply each man's ſtock into the time of its continuance, then ſay,

As the total ſum of all the products is to the whole gain or loſs;

So is each man's particular product, to his particular ſhare of the gain or loſs?

EXAMPLES.

1. A and B hold a piece of ground in common, for which they are to pay 36*l.* A put in 23 oxen for 27 days, and B 21 oxen for 35 days; what ought each man to pay of the rent?

$$\begin{aligned} 23 \times 27 &= 621 \\ 21 \times 35 &= \underline{735} \\ & \quad 1356 \end{aligned}$$

* Mr. Malcolm, Mr. Ward, and ſeveral other authors, have given an analytical inveſtigation of this rule; but the moſt general and elegant method I have met with is that by Mr, Hutton in p. 88 of his arithmetic, viz.

When the times are equal, the ſhares of the gain or loſs are evidently as the ſtocks, as in Single Fellowſhip; and when the ſtocks are equal the ſhares are as the times; wherefore when neither are equal, the ſhares muſt be as their products.

1356 : 36 :: 621

```
        621
        ---
         36
        72
      216
      ---------
1356)22356(16 l.
      1356
      ----
      8796
      8136
      ----
       660
        20
      ------
1356)13200(9 s.
      12204
      -----
        996
         12
      ------
1356)11952(8 d.
      10848
      -----
       1104
          4
       -----
1356)4416(3/4
      4068
      ----
       348
```

1356 : 36 :: 735

```
            735
          -----
            180
           108
          252
         -----
    1356)26460(19 l.
         1356
         -----
         12900
         12204
         -----
           696
            20
         -----
     1356)13920(10 s.
          1356
          -----
            360
             12
          -----
      1356)4320(3 d.
           4068
           -----
            252
              4
           -----
           1008
```

16 *l.*	9 *s.*	8¾ *d.* $\frac{348}{1356}$.	= *A's share.*
19 *l.*	10 *s.*	3 $\frac{1008}{1356}$.	= *B's share.*
36 *l.*	0 *s.*	0 *d.*	*the Proof.*

2. A

2. A, B and C hold a paſture in common, for which they pay 30*l.* *per annum.* A put into it 7 oxen for 3 months, B 9 oxen for 5 months, and C 4 for 12 months: what muſt each pay of the rent?

Anſ. A 5*l.* 10*s.* $6\frac{1}{4}$*d.* $\frac{30}{114}$. B 11*l.* 16*s.* 10*d.* $\frac{48}{114}$ *and* C 12*l.* 12*s.* $7\frac{1}{2}$*d.* $\frac{36}{114}$.

3. Three graziers hired a piece of land for 60*l.* 10*s.* A put in 5 ſheep for $4\frac{1}{2}$ months, B put in 8 for 5 months, and C put in 9 for $6\frac{1}{2}$ months: how much muſt each pay of the rent? *Anſ.* A 11*l.* 5*s.* B 20*l.* *and* C 29*l.* 5*s.*

Two merchants enter into partnerſhip for 18 months; A put into ſtock at firſt 200*l.* and at 8 months end he put in 100*l.* more, B put in at firſt 550 *l.* and at 4 months end took out 140 *l.* Now at the expiration of the time they find they have gained 526 *l.*: what is each man's juſt ſhare?

Anſ. A 192*l.* 19*s.* 0*d.* $\frac{672}{1254}$. B 333*l.* 0*s.* $11\frac{3}{4}$*d.* $\frac{582}{1254}$.

5. A with a capital of 1000*l.* began trade January 1ſt, 1776, and meeting with ſucceſs in buſineſs he took in B as a partner, with a capital of 1500*l.* on the 1ſt of March following. Three months after that they admit C as a third partner, who brought into ſtock 2800*l.* and after trading together till the firſt of the next year, they find there has been gained, ſince A's commencing of buſineſs, 1776*l.* 10*s.*: how muſt this be divided amongſt the partners? *Anſ.* A 457*l.* 9*s.* $4\frac{1}{4}$*d.* B 571*l.* 16*s.* $8\frac{1}{4}$*d.* C 747*l.* 3*s.* $11\frac{1}{4}$*d.*

ALLIGATION.

Alligation teacheth how to mix ſeveral ſimples of different qualities, ſo that the compoſition may be of a middle quality; and is commonly diſtinguiſhed into two principal caſes, called Alligation medial and Alligation alternate.

AL-

ALLIGATION MEDIAL.

Alligation medial is the method of finding the rate of the compound, from having the rates and quantities of the several simples given.

RULE.*

Multiply each quantity by its rate; then divide the sum of the products by the sum of the quantities, or the whole composition, and the quotient will be the rate of the compound required.

EXAMPLES.

1. Suppose 15 bushels of wheat at 5*s.* *per* bushel, and 12 bushels of rye at 3*s.* 6*d.* *per* bushel were mixed together: how must the compound be sold *per* bushel without loss or gain?

---

* The truth of this rule is too evident to need a demonstration.

Note, If an ounce or any other quantity of pure gold be reduced into 24 equal parts, these parts are called caracts; but gold is often mixed with some baser metal, which is called the alloy, and the mixture is said to be of so many caracts fine, according to the proportion of pure gold contained in it: thus, if 22 caracts of pure gold and 2 of alloy are mixed together, it is said to be 22 caracts fine.

If any one of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

60	42	15
15	12	12
300	504	27
60	900	
900	27)1404(52*d.*=4*s.* 4*d. the anſwer.*	
	135	
	54	
	54	

2. A compoſition being made of 5*lb.* of tea at 7*s.* *per lb.* 9*lb.* at 8*s.* 6*d.* *per lb.* and $14\frac{1}{2}$*lb.* at 5*s.* 10*d.* *per lb.* what is a *l b.* of it worth? *Anſ.* 6*s.* $10\frac{1}{2}$*d.*

3. Mixed 4 gallons of wine at 4*s.* 10*d.* *per gall.* with 7 gallons at 5*s.* 3*d.* *per gall.* and $9\frac{3}{4}$ gallons at 5*s.* 8*d.* *per gall.* what is a gallon of this compoſition worth? *Anſ.* 5*s.* $4\frac{1}{4}$*d.*

4. A mealman would mix 3 buſhels of flour at 3*s.* 5*d* *per* buſhel, 4 buſhels at 5*s.* 6*d.* *per* buſhel, and 5 buſhels at 4*s.* 8*d.* *per* buſhel: what is the worth of a buſhel of this mixture? *Anſ.* 4*s.* $7\frac{1}{2}$*d.*

5. A farmer mixes 20 buſhels of wheat at 5*s.* *per* buſhel, and 36 buſhels of rye at 3*s.* *per* buſhel, and 40 buſhels of barley at 2*s.* *per* buſhel: what is the worth of a buſhel of this mixture? *Anſ.* 3*s.*

6. A goldſmith melts 8 *lb.* $5\frac{1}{2}$ *oz.* of gold bullion of 14 caracts fine, with 12 *lb.* $8\frac{1}{2}$ *oz.* of 18 caracts fine: how many caracts fine is this mixture?

Anſ. $16\frac{204}{308}$ *caracts.*

7. A refiner melts 10 *lb.* of gold of 20 caracts fine with 16*lb.* of 18 caracts fine; how much alloy muſt he put to it to make it 22 caracts fine.

Anſ. It is not fine enough by $3\frac{6}{26}$ *caracts, ſo that no alloy muſt be put to it, but more gold.*

AL-

ALLIGATION ALTERNATE.

Alligation Alternate is the method of finding what quantity of any number of ſimples, whoſe rates are given, will compoſe a mixture of a given rate; ſo that it is the reverſe of alligation medial, and may be proved by it.

RULE 1.*

1. Write the rates of the ſimples in a column under each other.

2. Connect or link with a continued line, the rate of each ſimple which is leſs than that of the compound, with one, or any number, of thoſe that are greater than the compound; and each greater rate with one or any number of the leſs.

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* *Demon.* By connecting the leſs rate to the greater, and placing the differences between them and the mean rate alternately, the quantities reſulting are ſuch, that there is preciſely as much gained by one quantity as is loſt by the other, and therefore the gain and loſs upon the whole is equal, and is exactly the propoſed rate: and the ſame will be true of any other two ſimples managed according to the rule.

In like manner, let the number of ſimples be what they will, and with how many ſoever every one is linked, ſince it is always a leſs with a greater than the mean price, there will be an equal balance of loſs and gain between every two, and conſequently an equal balance on the whole. *Q. E. D.*

It is obvious, from the rule, that queſtions of this ſort admit of a great variety of anſwers; for, having found one anſwer we may find as many more as we pleaſe, by only multiplying or dividing each of the quantities found by 2, 3, or 4, &c. the reaſon of which is evident; for, if two quantities, of two ſimples, make a balance of loſs and gain, with reſpect to the mean price, ſo muſt alſo the double or treble, the $\frac{1}{2}$ or $\frac{1}{3}$ part, or any other ratio of theſe quantities, and ſo on *ad infinitum*.

Theſe kind of queſtions are called by algebraiſts *indeterminate* or *unlimited* problems, and by an analytical proceſs, theorems may be raiſed that will give all the *poſſible* anſwers.

3. Write

3. Write the difference between the mixture rate, and that of each of the simples, opposite the rates with which they are linked.

4. Then if only one difference stand against any rate, it will the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAMPLES.

1. A merchant would mix wines at 14*s.* 19*s.* 15*s.* and 22*s.* *per* gallon, so as that the mixture may be worth 18*s.* the gallon: what quantity of each must be taken.

18	14	4	*at* 14*s.*
	15	1	*at* 15*s.*
	19	3	*at* 19*s.*
	22	4	*at* 22*s.*

Or thus,

18	14	1+4	5 *at* 14*s.*
	15	1	1 *at* 15*s.*
	19	4+3	7 *at* 19*s.*
	22	4	4 *at* 22*s.*

2. How much wine at 6*s.* *per* gallon, and at 4*s.* *per* gallon, must be mixed together, that the composition may be worth 5*s.* *per* gallon? *Ans.* 1 *qt.* *or* 1 *gall.* &c.

3. How much corn at 2*s.* 6*d.* 3*s.* 8*d.* 4*s.* and 4*s.* 8*d.* *per* bushel, must be mixed together, that the compound may be worth 3*s.* 10*d.* *per* bushel? *Anſ.* 12 *at* 2*s.* 6*d.* 12 *at* 3*s.* 8*d.* 18 at 4*s.* *and* 18 *at* 4*s.* 8*d.*

4. A

4. A goldſmith has gold of 17, 18, 22, and 24 caracts fine: how much muſt he take of each to make it 21 caracts fine? *Anſ.* 3 *of* 17, 1 *of* 18, 3 *of* 22, *and* 4 *of* 24.

5. It is required to mix brandy at 8*s.* wine at 7*s.* cyder at 1 *s.* and water at 0 *per* gallon together, ſo that the mixture may be worth 5*s. per* gallon?

Anſ. 9 *galls. of brandy,* 9 *of wine,* 5 *of cyder, and* 5 *of water.*

RULE 2.*

When the whole compoſition is limited to a certain quantity.

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* A great number of queſtions might be here given relating to the ſpecific gravities of metals, &c. but as they are beſt performed by fractions. I ſhall only give one of the moſt curious, and work out the example at large.

Heiro, king of Syracuſe, gave orders for a crown to be made him entirely of pure gold: but ſuſpecting the workman had debaſed it by mixing it with ſilver or copper, he recommended the diſcovery of the fraud to the famous Archimedes; and deſired to know the exact quantity of alloy in the crown.

Archimedes, in order to detect the impoſition, procured two other maſſes, the one of pure gold, the other of ſilver or copper, and each of the ſame weight with the former; and by putting each ſeparately into a veſſel full of water, the quantity of water expelled by them determined their ſpecific bulks: from which and their given weights, the exact quantities of gold and alloy in the crown may be determined.

Suppoſe the weight of each crown to be 10 *lb.* and that the water expelled by the copper or ſilver was .92 *lb.* by the gold .52 *lb.* and by the compound crown .64 *lb.* what will be the quantities of gold and alloy in the crown?

The rates of the ſimples are 92 and 52, and of the compound 64; therefore

64	92	12 of copper
	52	28 of gold

And the ſum of theſe is 12+28=40, which ſhould have been but 10; whence, by the rule,

40 : 10 :: 12 : 3 *lb.* of copper } the anſwer
40 : 10 :: 28 : 7 *lb.* of gold }

Find an anſwer as before by linking; then ſay, as the ſum of the quantities, or differences thus determined, is to the given quantity, ſo is each ingredient, found by linking, to the required quantity of each.

EXAMPLES.

1. How many gallons of water at 0*s. per* gallon muſt be mixed with wine worth 3*s. per* gallon, ſo as to fill a veſſel of 100 gallons, and that a gallon may be afforded at 2*s.* 6*d.*?

$$30\left\{\begin{array}{l} 0 \\ 36 \end{array}\right. \quad \begin{array}{r} 6 \\ 30 \\ \hline 36 \end{array}$$

$36 : 100 :: 6 \qquad 36 :: 100 :: 30$

$$\begin{array}{r} 100 \\ 6 \\ \hline 36)600(16 \\ 36 \\ \hline 240 \\ 216 \\ \hline 24 \end{array} \qquad \begin{array}{r} 100 \\ 30 \\ \hline 36)3000(83 \\ 288 \\ \hline 120 \\ 108 \\ \hline 12 \end{array}$$

Anſ. $83\frac{12}{36}$ *gallons of wine, &* $16\frac{24}{36}$ *of water.*

2. A grocer has currants at 4*d.* 6*d.* 9*d.* and 11*d. per lb.* and he would make a mixture of 240 *lb.* ſo that it might be afforded at 8*d. per lb.* how much of each ſort muſt he take?

Anſ. 72 *lb. at* 4*d*, 24 *at* 6*d*, 48 *at* 9*d*, *and* 96 at 11*d.*

3. How

3. How much gold of 15, of 17, of 18, and of 22 caracts fine, must be mixed together to form a composition of 40 oz. of 20 caracts fine?

Ans. 5 *oz. of* 15, *of* 17 *and of* 18, *and* 25 *oz. of* 22.

RULE 3.*

When one of the ingredients is limited to a certain quantity.

Take the difference between each price and the mean rate as before; then,

As the difference of that simple, whose quantity is given, is to the rest of the differences severally, so is the quantity given to the several quantities required.

EXAMPLES.

1. How much wine at 5 *s.* at 5 *s.* 6 *d.* and 6*s.* the gallon must be mixed with 3 gallons at 4*s.* *per gallon*, so that the mixture may be worth 5 *s.* 4*d.* *per gallon*?

$$64\left\{\begin{array}{l}48\\60\\66\\72\end{array}\right.\qquad\begin{array}{l}8+2=10\\8+2=10\\16+4=20\\16+4=20\end{array}$$

10 : 10 :: 3 : 3

10 : 20 :: 3 : 6

10 : 20 :: 3 : 6

Ans. 3 *gallons at* 5*s.* 6 *at* 5*s* 6*d.* *and* 6 *at* 6*s.*

---

* In the very same manner questions may be wrought when several of the ingredients are limited to certain quantities, by finding first for one limit and then for another.

The two last rules can want no demonstration, as they evidently result from the first, the reason of which has been already explained.

2. A grocer would mix teas at 12 *s.* 10 *s.* and 6 *s.* with 20 *lb.* at 4 *s. per lb.* how much of each sort must he take to make the composition worth 8 *s. per lb.*

Ans. 20*lb. at* 4*s.* 10*lb. at* 6*s.* 10*lb. at* 10*s. and* 20*lb. at* 12*s.*

3. How much gold of 15 of 17 and of 22 caracts fine, must be mixed with 5 *oz.* of 18 caracts fine, so that the composition may be 20 caracts fine?

Ans. 5 *oz. of* 15 *caracts fine,* 5 *oz. of* 17, *and* 25 *of* 22.

VULGAR FRACTIONS.

Fractions, or broken numbers, are expressions for any assignable parts of an unit; and are represented by two numbers, placed one above the other, with a line drawn between them.

The figure above the line is called the *numerator*, and that below the line the *denominator*.

The denominator shews how many parts the integer is divided into, and the numerator shews how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed.

1. A *proper fraction* is when the numerator is less then the denominator, as $\frac{3}{4}$, $\frac{4}{5}$, ^{5}, &c.

2. An *improper fraction* is when the numerator exceeds the denominator, as $\frac{8}{3}$, $\frac{110}{21}$, &c.

3. A *single fraction* is a simple expression for any number of parts of the integer.

4. A *compound fraction* is the fraction of a fraction, as $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{5}{6}$, &c.

5. A *mixed number* is composed of a whole number and a fraction, as $8\frac{1}{5}$, $17\frac{6}{13}$, &c.

Note, any whole number may be expressed like a fraction by writing 1 underneath it.

6. The *common measure* of two or more numbers, is that number which will divide each of them without a remainder.

remainder. Thus 3 is the common measure of 12 and 15; and the *greatest* number that will do this is called the *greatest common measure.*

7. A number which can be measured by two or more numbers, is called their *common multiple*; and if it be the *least* number which can be so measured, it is called their *least common multiple*; thus 30, 45, 60 and 75, are multiples of 3 and 5; but their least common multiple is 15. *

PROBLEM I.

To find the greatest common measure of two or more numbers.

RULE. †

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on,

* A *prime number* is that which can only be measured by an unit.

That number which is produced by multiplying several numbers together, is called a *composite number.*

A *perfect number* is equal to the sum of all its aliquot parts.

The following perfect numbers are taken from the Petersburg acts, and are all that are known at present.

6
28
496
8128
33550336
8589869056
137438691328
2305843008139952128
2417851639228158837784576
9903520314282971830448816128

There are several other numbers which have received different denominations, but they are principally of use in Algebra, and the higher parts of the mathematics.

† This and the following problem will be found very useful in the doctrine of fractions, and several other parts of Arithmetic.

on, always dividing the laſt diviſor by the laſt remainder, till nothing remains, then will the laſt diviſor be the greateſt common meaſure required.

2. When there are more than two numbers, find the greateſt common meaſure of two of them as before; and of that common meaſure and one of the other numbers; and ſo on, through all the numbers to the laſt; then will the greateſt common meaſure laſt found be the anſwer.

3. If 1 happens to be the common meaſure, the given numbers are prime to each other, and found to be incommenſurable.

EXAMPLES.

1. Required the greateſt common meaſure of 918, 1998, and 522.

The truth of the rule may be ſhewn from the 1ſt. example. For ſince 54 meaſures 108, it alſo meaſures 108 + 54, or 162.

Again, ſince 54 meaſures 108, and 162, it alſo meaſures $\overline{5 \times 162 + 108}$ or 918. In the ſame manner it will be found to meaſure $\overline{2 \times 918} + 162$ or 3698, and ſo on. Therefore 84 meaſures both 918 and 1998.

It is alſo the greateſt common meaſure; for ſuppoſe there be a greater; then ſince the greater meaſures 918 and 1998, it alſo meaſures the remainder 162; and ſince it meaſures 162 and 918, it alſo meaſures the remainder 108; in the ſame manner it will be found to meaſure the remainder 54; that is, the greater meaſures the leſs, which is abſurd. Therefore 54 is the greateſt common meaſure.

In the very ſame manner the demonſtration may be applied to 3 or more numbers.

918) 1998(2
   1836
   ———
   162)918(5
      810
      ———
      108)162(1
         108
         ———
         54)108(2
            108
            ———

So 54 *is the greatest common measure of* 1998 *and* 918

Hence 54)522(9
   486
   ———
   36)54(1
      36
      ——
      18)36(2
         36
         ——

Therefore 18 *is the answer required.*

2. What is the greatest common measure of 612 and 540? *Ans.* 36

3. What is the greatest common measure of 720, 336 and 1736? *Ans.* 8

PROBLEM 2.

To find the least common multiple of two or more numbers.

RULE.*

1. Divide by any number that will divide two or more of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2. Divide the second line as before, and so on till there are no two numbers that can be divided; then the continued product of the divisors and quotients will give the multiple required.

* The reason of this rule, may, also, be shewn from the 1st. example, thus: it is evident, that $3 \times 5 \times 8 \times 10 = 1200$ may be divided by 3, 5, 8 and 10, without a remainder; but 10 is a multiple of 5, therefore $3 \times 5 \times 8 \times 2$, or 240, is, also, divisible by 3, 5, 8 and 10. Also 8 is a multiple of 2; therefore $3 \times 5 \times 4 \times 2 = 120$ is also divisible by 3, 5, 8 and 10; and is, evidently, the least number that can be so divided.

EXAM-

EXAMPLES.

1. What is the least common multiple of 3, 5, 8 and 10?

5)3, 5, 8, 10

2)3, 1, 8, 2

3, 1, 4, 1

$5 \times 2 \times 3 \times 4 = 120$ *the answer.*

2. What is the least common multiple of 4 and 6? *Ans.* 12

3. What is the least number that 3, 4, 8 and 12 will measure? *Ans.* 24

4. What is the least number that can be divided by the nine digits, without a remainder? *Ans.* 2520

REDUCTION OF VULGAR FRACTIONS.

Reduction of Vulgar Fractions is the bringing them out of one form into another, in order to prepare them for the operations of addition, subtraction, &c.

CASE 1.

To abbreviate or reduce fractions to their lowest terms.

RULE.*

Divide the terms of the given fraction by any number that will divide them without a remainder, and these

 quotients

* That dividing both the terms of the fraction, equally, by any number whatever, will give another fraction equal to the former, is evident. And if those divisions are performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note. 1. Any number ending with an even number, or a cypher, is divisible by 2.

2. Any number ending with 5, or 0, is divisible by 5.

9. If

quotients again in the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms.

Or,

Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{144}{240}$ to its lowest terms.

$$\overset{(2)}{\frac{144}{240}} = \overset{(2)}{\frac{72}{120}} = \overset{(3)}{\frac{36}{60}} = \overset{(2)}{\frac{12}{20}} = \overset{(2)}{\frac{6}{10}} = \frac{3}{5}, \text{ the answer.}$$

Or thus.

```
144) 240(1
     144
     ---
      96) 144(1
           96
           ---
           48) 96(2
               96
               --
```

Therefore 48 *is the greatest common measure, and* 48) $\frac{144}{240} = \frac{3}{5}$ *the same as before.*

2. Re-

3. If the right-hand place of any number be 0, the whole is divisible by 10.

4. If the two right-hand figures of any number are divisible by 4, the whole is divisible by 4.

5. If the three right-hand figures of any number are divisible by 8, the whole is divisible by 8.

6. If

2. Reduce $\frac{48}{272}$ to its least terms. Ans. $\frac{3}{17}$
3. Bring $\frac{192}{576}$ to its lowest terms. Ans. $\frac{1}{3}$
4. Reduce $\frac{825}{960}$ to its least terms. Ans. $\frac{55}{64}$
5. Reduce $\frac{252}{364}$ to its lowest terms. Ans. $\frac{9}{13}$
6. Reduce $\frac{5184}{6912}$ to its least terms. Ans. $\frac{3}{4}$
7. Reduce $\frac{1344}{1536}$ to its lowest terms. Ans. $\frac{7}{8}$
8. Abbreviate $\frac{6896800}{36700160}$ as much as possible.

Ans. $\frac{43105}{229376}$

CASE 2.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator to the product, then that sum written above the denominator will form the fraction required.

---

6. If the sum of the digits constituting any number be divisible by 3, or 9, the whole is divisible by 3, or 9.

7. If a number cannot be divided by some number less than the square root thereof, that number is a prime.

8. All prime numbers, except 2 and 5, have 1, 3, 7 or 9 in the place of units; and all other numbers are composite.

9. When numbers, with the sign of addition or subtraction between them, are to be divided by any number, each of the numbers must be divided. Thus $\frac{4+8+10}{2} = 2+4+5 = 11$.

10. But if the numbers have the sign of multiplication between them, only one of them must be divided. Thus $\frac{3\times8\times10}{2\times6} =$ $\frac{3\times4\times10}{1\times6} = \frac{1\times4\times10}{1+2} = \frac{1\times2\times10}{1\times1} = \frac{20}{1} = 20$.

* All fractions represent a division of the numerator by the denominator, and are taken altogether as proper and adequate expressions for the quotient. Thus the quotient of 2 divided by 3 is $\frac{2}{3}$; from whence the rule is manifest; for if any number is multiplied and divided by the same number, it is evident the quotient must be the same as the quantity first given.

EXAMPLES.

1. Reduce $27\frac{2}{9}$ to its equivalent improper fraction.

$$\begin{array}{r} 27 \\ 9 \\ \hline 243 \\ 2 \\ \hline 245 \\ \hline 9 \end{array}$$

Or $\dfrac{\overline{27 \times 9} + 2}{9} = \frac{245}{9}$ *the answer.*

2. Reduce $183\frac{5}{21}$ to its equivalent improper fraction. *Ans.* $\frac{3848}{21}$

3. Reduce $514\frac{5}{16}$ to an improper fraction. *Ans.* $\frac{8229}{16}$

4. Reduce $100\frac{19}{59}$ to an improper fraction. *Ans.* $\frac{5919}{59}$

5. Reduce $47\frac{3147}{8400}$ to an improper fraction. *Ans.* $\frac{397947}{8400}$

CASE 3.

To reduce an improper fraction to its equivalent whole or mixed number.

RULE.*

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

EXAMPLES.

1. Reduce $\frac{981}{16}$ to its equivalent whole or mixed number.

* This rule is plainly the reverse of the former, and has its reason in the nature of common division.

16)

$$\begin{array}{r} 16)981(61\frac{5}{16} \\ \underline{96} \\ 21 \\ \underline{16} \\ 5 \end{array}$$

Or

$$\frac{981}{16} = 981 \div 16 = 61\tfrac{5}{16} \textit{ the answer.}$$

2. Reduce $\frac{56}{8}$ to its equivalent whole or mixed number. *Ans.* 7

3. Reduce $\frac{1245}{22}$ to its equivalent whole or mixed number. *Ans.* $56\frac{13}{22}$

4. Reduce $\frac{3848}{21}$ to its equivalent whole or mixed number. *Ans.* $183\frac{5}{21}$

5. Reduce $\frac{621613}{514}$ to its equivalent whole or mixed number. *Ans.* $1209\frac{187}{514}$

CASE 4.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.*

Multiply the whole number by the given denominator, and place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 7 to a fraction whose denominator shall be 9.

$7 \times 9 = 63$, *and* $\frac{63}{9}$ *the answer.*

And $\frac{63}{9} = 63 \div 9 = 7$ *the proof.*

* Multiplication and division are here equally used, and consequently the result is the same as the quantity first proposed.

2. Reduce 13 to a fraction whose denominator shall be 12. *Anſ.* $\frac{156}{12}$

3. Reduce 100 to a fraction whose denominator shall be 90. *Anſ.* $\frac{9000}{90}$

CASE 5.

To reduce a compound fraction to an equivalent ſimple one.

RULE*

Multiply all the numerators together for a numerator, and all the denominators together for the denominator, and they will form the ſimple fraction required.

If part of the compound fraction be a whole or mixed number, it muſt be reduced to a fraction by one of the former caſes.

When it can be done, divide any two terms of the fraction by the ſame number, and uſe the quotients inſtead thereof.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{8}{11}$ to a ſimple fraction.

$$\frac{2 \times 3 \times 4 \times 8}{3 \times 4 \times 5 \times 11} = \frac{192}{660} = \frac{16}{55} \text{ the anſwer.}$$

Or

$$\frac{2 \times \cancel{3} \times \cancel{4} \times 8}{\cancel{3} \times \cancel{4} \times 5 \times 11} = \frac{16}{55} \text{ as before.}$$

---

* That a compound fraction may be repreſented by a ſimple one is very evident, ſince a part of a part muſt be equal to ſome part of the whole. The truth of the rule for this reduction may be ſhewn as follows.

Let the compound fraction to be reduced be $\frac{2}{3}$ of $\frac{4}{7}$. Then $\frac{1}{3}$ of $\frac{4}{7} = \frac{4}{7} \div 3 = \frac{4}{21}$, and conſequently $\frac{2}{3}$ of $\frac{4}{7} = \frac{4}{21} \times 2 = \frac{8}{21}$ the ſame as by the rule, and the like will be found to be true in all caſes.

If the compound fraction conſiſts of more numbers than 2, the two firſt may be reduced to one, and that one and the third will be the ſame as a fraction of two numbers; and ſo on.

2. Reduce

2. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a simple fraction. *Anf.* $\frac{1}{4}$

3. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{11}$ to a simple fraction. *Anf.* $\frac{5}{33}$

4. Reduce $\frac{11}{12}$ of $\frac{7}{13}$ of $\frac{8}{19}$ of 10 to a simple fraction. *Anf.* $\frac{1540}{741}$

CASE 6.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

RULE 1.*

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators continually for a common denominator.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{4}{7}$ to equivalent fractions, having a common denominator.

$1 \times 5 \times 7 = 35$ *the new numerator for* $\frac{1}{2}$.
$3 \times 2 \times 7 = 42$ *ditto for* $\frac{3}{5}$,
$4 \times 2 \times 5 = 40$ *ditto for* $\frac{4}{7}$,
$2 \times 5 \times 7 = 70$ *the common denominator.*

Therefore the new equivalent fractions are $\frac{35}{70}$, $\frac{42}{70}$, *and* $\frac{40}{70}$, *the anfwer.*

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$ to fractions, having a common denominator. *Anf.* $\frac{144}{288}$, $\frac{192}{288}$, $\frac{240}{288}$, $\frac{252}{288}$

---

* By placing the numbers multiplied, properly under one another, it will be feen that the numerator and denominator of every fraction are multiplied by the very fame number, and confequently their values are not altered. Thus in the firft example;

$$\frac{1}{2}\left|\frac{\times 5 \times 7}{\times 5 \times 7}\right. \quad \frac{3}{5}\left|\frac{\times 2 \times 7}{\times 2 \times 7}\right. \quad \frac{4}{7}\left|\frac{\times 2 \times 5}{\times 2 \times 5}\right.$$

In the 2d. rule, the common denominator is a multiple of all the denominators, and confequently will divide by any of them; it is manifeft therefore that proper parts may be taken for all the numerators as required.

3. Reduce $\frac{1}{3}$, $\frac{3}{4}$ of $\frac{4}{5}$, $5\frac{1}{2}$ and $\frac{2}{19}$ to a common denominator. *Ans.* $\frac{190}{570}$, $\frac{342}{570}$, $\frac{3135}{570}$, $\frac{60}{570}$

4. Reduce $\frac{11}{13}$, $\frac{3}{4}$ of $1\frac{1}{4}$, $\frac{9}{11}$ and $\frac{5}{7}$ to a common denominator. *Ans.* $\frac{13552}{16016}$, $\frac{15015}{16016}$, $\frac{13104}{16016}$, $\frac{11440}{16016}$

RULE 2.

To reduce any given fractions to others, which ſhall have the leaſt common denominator.

1. Find the leaſt common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$ to fractions, having the leaſt common denominator poſſible.

$$\begin{array}{r|l} 3 & 2.\ 3.\ 6 \\ \hline 2 & 2.\ 1.\ 2 \\ \hline \end{array}$$

$1 \times 1 \times 1 \times 2 \times 3 = 6 =$ *leaſt common denom.*

$6 \div 2 \times 1 = 3$ *the 1ſt numerator*; $6 \div 3 \times 2 = 4$ *the 2d. numerator*; $6 \div 6 \times 5 = 5$ *the 3d. numerator.*

Whence the required fractions are, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$.

2. Reduce $\frac{7}{12}$ and $\frac{11}{18}$ to fractions, having the leaſt common denominator. *Anſ.* $\frac{21}{36}$, $\frac{22}{36}$

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to the leaſt common denominator. *Anſ.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$

4. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$ and $\frac{7}{10}$ to the leaſt common denominator. *Anſ.* $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{63}{90}$

5. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$ to equivalent fractions having the leaſt common denominator poſſible.

Anſ. $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$, $\frac{34}{48}$

CASE 7.

To find the value of a fraction in the known parts of the integer.

RULE.*

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remains, multiply it by the next inferior denomination, and divide by the denominator as before; and so on as far as necessary; and the quotients placed after one another, in their order, will be the answer required.

EXAMPLES.

1. What is the value of $\frac{5}{7}$ of a shilling?

$$\begin{array}{r} 5 \\ 12 \\ \hline 7)60 \\ \hline 8\text{-}4 \\ 4 \\ \hline 7)16 \\ \hline 2\text{-}2 \end{array}$$

Ans. 8½*d.* $\frac{2}{7}$

2. What is the value of $\frac{3}{8}$ of a pound sterling? *Ans.* 7*s.* 6*d.*

3. What is the value of $\frac{2}{9}$ of a guinea? *Ans.* 4*s.* 8*d.*

4. What is the value of $\frac{4}{7}$ of half a crown? *Ans.* 1*s.* $5\frac{1}{7}$*d.*

---

* The numerator of a fraction may be considered as a remainder, and the denominator as a divisor; therefore this rule has its reason in the nature of compound division, and the valuation of remainders in the rule of three, which has been already sufficiently explained.

5. What is the value of $\frac{13}{19}$ of a moidore?

Ans. 18*s.* $5\frac{13}{19}$*d.*

6. What is the value of $\frac{3}{5}$ of a pound troy?

Ans. 7*oz.* 4*dwts.*

7. What is the value of $\frac{4}{7}$ of a pound avoirdupois?

Ans. 9 *oz.* $2\frac{2}{7}$*dr.*

8. What is the value of $\frac{7}{9}$ of a *cwt.*

Ans. 3*qr.* 3*lb.* 10*oz.* $12\frac{4}{9}$ *dr.*

9. What is the value of $\frac{3}{17}$ of a mile?

Ans. 1 *fur.* 16 *po.* 2 *yds.* 1 *foot* $9\frac{3}{17}$ *in.*

10. What is the value of $\frac{5}{9}$ of an ell english?

Ans. 2 *qr.* $3\frac{1}{9}$ *na.*

11. What is the value of $\frac{5}{8}$ of an acre?

Ans. 2 *ro.* 20 *po.*

12. What is the value of $\frac{7}{8}$ of a tun of wine?

Ans. 3 *hhds.* 31 *gall.* 2 *qr.*

13. What is the value of $\frac{2}{15}$ of a *hhd.* of ale?

Ans. 6 *gall.* $3\frac{1}{15}$ *pi.*

14. What is the value of $\frac{5}{9}$ of a quarter of corn?

Ans. 4 *bu.* 1 *pe.* 1 *ga.* $2\frac{2}{9}$ *qr.*

15. What is the value of $\frac{7}{13}$ of a day?

Ans. 12 *ho.* 55 *min.* $23\frac{1}{13}$ *sec.*

CASE 8.

To reduce a fraction of one denomination to that of another, retaining the same value.

RULE.*

Make a compound fraction of it, and reduce it to a simple one.

---

* The reason of this practice is explained in the rule for reducing compound fractions to single ones.

The rule might have been distributed into 2 or 3 different cases, but the directions here given may very easily be applied to any question that can be proposed in those cases, and will be more easily understood by an example or two, than by a multiplicity of words. Let there be taken one question in each of the cases.

EXAMPLES.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.

$\frac{5}{6}$ *of* $\frac{1}{12}$ *of* $\frac{1}{20} = \frac{5}{1440} = \frac{1}{288}$ *the answer.*

and $\frac{1}{288}$ *of* $\frac{20}{1}$ *of* $\frac{12}{1} = \frac{240}{288} = \frac{5}{6}$ *the proof.*

2. Reduce $\frac{2}{3}$ of a farthing to the fractions of a pound. *Ans.* $\frac{1}{1440}$

3. Reduce $\frac{1}{18}$ *l.* to the fraction of a penny. *Ans.* $\frac{40}{3}$

4. Reduce $\frac{4}{5}$ of a *dwt.* to the fraction of a pound troy. *Ans.* $\frac{1}{300}$

5. Reduce $\frac{6}{7}$ of a pound avoirdupois to the fraction of *cwt.* *Ans.* $\frac{6}{784}$

6. Reduce $\frac{9}{6552}$ of a *hhd.* of wine to the fraction of a pint. *Ans.* $\frac{9}{13}$

7. Reduce $\frac{3}{13}$ of a month to the fraction of a day. *Ans.* $\frac{84}{13}$

* 8. Reduce 7*s.* 3*d.* to the fraction of a pound. *Ans.* $\frac{29}{80}$

9. Express 6 *fur.* 16 *po.* in the fraction of a mile. *Ans.* $\frac{4}{5}$

† 10. Reduce $\frac{2}{7}$*l.* to the fraction of a guinea. *Ans.* $\frac{40}{147}$

11. Express $\frac{5}{8}$ of a crown in the fraction of a guinea. *Ans.* $\frac{25}{168}$

12. Express $\frac{5}{6}$ of a half crown in the fraction of a shilling. *Ans.* $\frac{25}{12}$

13. Express $\frac{6}{7}$ of a moidore in the fraction of a crown. *Ans.* $\frac{162}{35}$

Thus * 7*s.* 3*d.* = 87*d.* and 1*l.* = 240*d.* $\therefore \frac{87}{240} = \frac{29}{80}$ the answer.

† $\frac{2}{7}l. = \frac{2}{7}$ of $\frac{20}{1} = \dfrac{2 \times 20}{7 \times 1} = \frac{40}{7}s.$ and $\frac{40}{7}$ of $\frac{1}{21} =$

$\dfrac{40 \times 1}{7 \times 21} = \frac{40}{147}$ guinea, the answer.

ADDITION OF VULGAR FRACTIONS.

R U L E.*

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. Add $3\frac{5}{8}$, $\frac{7}{8}$, $\frac{4}{5}$ of $\frac{7}{8}$ and 7 together.

First $3\frac{5}{8} = \frac{29}{8}$, $\frac{\cancel{4}}{5}$ *of* $\frac{7}{\underset{2}{\cancel{8}}} = \frac{7}{10}$, $7 = \frac{7}{1}$

Then the fractions are $\frac{29}{8}$, $\frac{7}{8}$, $\frac{7}{10}$, *and* $\frac{7}{1}$; $\therefore$

$$\begin{array}{r}
29 \times 8 \times 10 \times 1 = 2320 \\
7 \times 8 \times 10 \times 1 = 560 \\
7 \times 8 \times 8 \times 1 = 448 \\
7 \times 8 \times 8 \times 10 = 4480 \\
\hline
8 \times 8 \times 10 \times 1 = \dfrac{7808}{640} = 12\tfrac{128}{640} = 12\tfrac{1}{5} \textit{ the ans.}
\end{array}$$

2. Add $\frac{5}{8}$, $7\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{3}{4}$ together. *Ans.* $8\frac{3}{8}$

3. What is the sum of $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$, and $9\frac{3}{20}$? *Ans.* $10\frac{1}{60}$

---

* Fractions before they are reduced to a common denominator are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts of the same thing, their sum, or difference, may then be as properly expressed, by the sum or difference of the numerators as the sum or difference of any two quantities whatever, by the sum or difference of their individuals; whence the reason of the rules, both for addition and subtraction is manifest.

4. What

4. What is the sum of $\frac{9}{10}$ of 6 $\frac{7}{8}$, $\frac{4}{7}$ of $\frac{1}{2}$, and 7 $\frac{1}{2}$?
Ans. 13$\frac{109}{112}$

5. Add $\frac{1}{7}$ *l.* $\frac{2}{9}$ *s.* and $\frac{5}{12}$ of a penny together.
Ans. $\frac{3139}{1008}$, *or* 3 *s.* 1 *d.* 1 $\frac{10}{21}$

6. What is the sum of $\frac{2}{7}$ of 15 *l.* 3 $\frac{3}{7}$ *l.* $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of a *l.* and $\frac{2}{3}$ of $\frac{3}{7}$ of a *s.* *Ans.* 7 *l.* 17 *s.* 5 $\frac{4}{7}$ *d.*

7. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{3}{8}$ of a mile together. *Ans.* 1540 *yds.* 2 *feet*, 9 *in.*

8. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. *Ans.* 2 *da.* 14 $\frac{1}{2}$ *ho.*

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as in addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

EXAMPLES.

1. From $\frac{2}{3}$ take $\frac{2}{9}$ of $\frac{3}{7}$.

$$\frac{2}{\cancel{9}_{3}} \text{ of } \frac{\cancel{3}}{7} = \frac{2}{21}, \ \& \ \frac{2}{3} = \frac{14}{21}$$

$$\therefore \frac{14}{21} - \frac{2}{21} = \frac{12}{21} = \frac{4}{7} \textit{ the answer required.}$$

2. From $\frac{97}{100}$ take $\frac{3}{7}$. *Ans.* $\frac{379}{700}$
3. From 96 $\frac{1}{3}$ take 14 $\frac{3}{7}$. *Ans.* 81 $\frac{19}{21}$
4. From 14 $\frac{1}{4}$ take $\frac{2}{3}$ of 19. *Ans.* 1 $\frac{7}{12}$
5. From $\frac{1}{2}$ *l.* take $\frac{3}{4}$ *s.* *Ans.* 9*s.* 3*d.*
6. From $\frac{3}{5}$ *oz.* take $\frac{7}{8}$ *dwt.* *Ans.* 11 *dwts.* 3 *gr.*
7. From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile. *Ans.* 1 *mi.* 2 *fur.* 16 *po.*
8. From 7 weeks take 9 $\frac{7}{10}$ days. *Ans.* 5 *we.* 4 *da.* 7 *ho.* 12 *min.*

MULTIPLICATION OF VULGAR FRACTIONS.

R U L E.*

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators is the numerator, and the product of the denominators the denominator of the product required.

EXAMPLES.

1. Required the continued product of $2\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{3}$ of $\frac{5}{6}$, and 2.

$$2\tfrac{1}{2}, = \tfrac{5}{2}, \ \tfrac{1}{3} \text{ of } \tfrac{5}{6} = \frac{1 \times 5}{3 \times 6} = \tfrac{5}{18}, \text{ and } 2 = \tfrac{2}{1};$$

$$\textit{Then } \tfrac{5}{2} \times \tfrac{1}{8} \times \tfrac{5}{18} \times \tfrac{2}{1} = \frac{5 \times 1 \times 5 \times \cancel{2}}{\cancel{2} \times 8 \times 18 \times 1} = \tfrac{25}{144}$$

the answer.

2. Multiply $\frac{4}{15}$ by $\frac{5}{24}$. *Ans.* $\frac{1}{18}$
3. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$. *Ans.* $\frac{9}{16}$
4. Multiply $\frac{1}{2}$ of 7 by $\frac{3}{6}$. *Ans.* $1\frac{3}{4}$
5. Multiply $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$. *Ans.* $\frac{23}{84}$
6. Multiply $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$, and $18\frac{4}{5}$ continually together. *Ans.* $9\frac{9}{140}$
7. What is the continual product of $\frac{2}{3}$, $3\frac{1}{4}$, 5, and $\frac{3}{4}$ of $\frac{3}{5}$? *Ans.* $4\frac{7}{8}$
8. What is the continual product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$, and $4\frac{1}{6}$? *Ans.* $2\frac{8}{21}$

---

* Multiplication by a fraction implies the taking some part or parts of the multiplicand, and, therefore, may be truly expressed by a compound fraction. Thus $\frac{3}{4}$ multiplied by $\frac{5}{8}$ is the same as $\frac{3}{4}$ of $\frac{5}{8}$; and as the directions of the rule agree with the method already given to reduce these fractions to simple ones, it is shewn to be right.

DIVI-

DIVISION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as before; then invert the divisor, and proceed exactly as in multiplication.

EXAMPLES.

1. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$.

$$\frac{1}{5} \text{ of } 19 = \frac{1 \times 19}{5 \times 1} = \frac{19}{5}, \text{ and } \frac{2}{3} \text{ of } \frac{3}{4} = \frac{\cancel{2} \times \cancel{3}}{\cancel{3} \times \underset{2}{\cancel{4}}} = \frac{1}{2}$$

$$\therefore \frac{19}{5} \times \frac{2}{1} = \frac{19 \times 2}{5 \times 1} = \frac{38}{5} = 7\tfrac{3}{5} \textit{ the quotient required.}$$

2. Divide $\frac{4}{7}$ by $\frac{2}{3}$. *Ans.* $\frac{6}{7}$
3. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7. *Ans.* $2\frac{13}{21}$
4. Divide $3\frac{1}{6}$ by $9\frac{1}{2}$. *Ans.* $\frac{1}{3}$
5. Divide $\frac{7}{8}$ by 4. *Ans.* $\frac{7}{32}$
6. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $\frac{2}{3}$
7. Divide 5 by $\frac{7}{10}$. *Ans.* $7\frac{1}{7}$
8. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 91. *Ans.* $71\frac{1}{2}$

---

* The reason of the rule may be shewn thus. Suppose it were required to divide $\frac{3}{4}$ by $\frac{2}{5}$. Now $\frac{3}{4} \div 2$ is manifestly $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{4 \times 2}$; but $\frac{2}{5} = \frac{1}{5}$ of 2, $\therefore \frac{1}{5}$ of 2, or $\frac{2}{5}$ must be contained 5 times as often in $\frac{3}{4}$ as 2 is; that is $\frac{3 \times 5}{4 \times 2} =$ the answer; which is according to the rule; and will be so in all cases.

Note, A fraction is multiplied by an integer, by dividing the denominator by it, or multiplying the numerator. And divided by an integer, by dividing the numerator, or multiplying the denominator.

THE

THE RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.*

Make the necessary preparations as before directed, and invert the first term of the proportion; then multiply the three terms continually together, and the product will be the answer.

EXAMPLES.

1. If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$ of a *l.* what will $\frac{6}{15}$ of an English ell cost?

First $\frac{3}{5}$ *of a yard* $= \frac{3}{5}$ *of* $\frac{4}{1}$ *of* $\frac{1}{5} = \frac{3 \times 4 \times 1}{5 \times 1 \times 5} = \frac{12}{25}$ *of an ell.*

Then $\frac{12}{25}$ *ell* : $\frac{7}{12}$ *l.* :: $\frac{6}{15}$ *ell.*

$$And\ \frac{7}{12} \times \frac{6}{15} \times \frac{25}{12} = \frac{7 \times \cancel{6}^{\,} \times \cancel{25}^{5}}{\cancel{12}_{2} \times \cancel{15}_{3} \times 12} = \frac{35}{72}.$$

$= 9$ *s.* 8 *d.* $\frac{2}{3}$ *the answer.*

2. If $\frac{3}{5}$ of an ell of holland cost $\frac{1}{3}$ *l.* what will $12\frac{2}{3}$ ells cost? *Ans.* 7 *l.* 0 *s.* $8\frac{2}{3}$ *d.*

3. If $\frac{5}{7}$ *oz.* cost $\frac{11}{12}$ *l.* what will 1 *oz.* cost? *Ans.* 1 *l.* 5 *s.* 8 *d.*

4. If $\frac{3}{16}$ of a ship cost 273 *l.* 2 *s.* 6 *d.* what is $\frac{5}{32}$ of her worth? *Ans.* 227 *l.* 12 *s.* 1 *d.*

5. At $1\frac{1}{2}$ *l. per cwt.* what does $3\frac{1}{3}$ *lb.* come to? *Ans.* $10\frac{5}{7}$ *d.*

6. If $\frac{5}{8}$ of a gallon cost $\frac{5}{8}$ *l.* what will $\frac{5}{9}$ of a tun cost? *Ans.* 140 *l.*

---

* This rule depends upon the same principles as the rule of three in whole numbers.

7. A mercer bought 3 ½ pieces of silk, each containing 24 ⅓ yards, at 6 *s.* ½ *d. per yard*, what does the whole come to? *Ans.* 25 *l.* 14 *s.* 6 ½ *d.* ⅓.

8. Agreed for the carriage of 2 ½ tons of goods 2 $\frac{9}{10}$ miles for $\frac{3}{40}$ of a guinea, what is that *per cwt.* for a mile? *Ans.* $\frac{378}{725}$ *of a farthing.*

9. A person having ⅗ of a coal mine sells ¾ of his share for 171 *l.* what is the whole mine worth? *Ans.* 380 *l.*

THE RULE OF THREE INVERSE IN VULGAR FRACTIONS.

EXAMPLES.

1. What quantity of shalloon that is ¾ *yd.* wide, will line 9 ½ yards of cloth that is 2 ½ yards wide?

First 2 ½ *yds.* = $\frac{5}{2}$, & 9 ½ *yds.* = $\frac{19}{2}$.

Then $\frac{5}{2}$ *yds.* : $\frac{19}{2}$ *yds.* :: ¾ *yd.*

$$And\ \frac{5}{2} \times \frac{19}{2} \times \frac{4}{3} = \frac{5 \times 19 \times \cancel{4}}{\cancel{2} \times \cancel{2} \times 3} = \frac{95}{3}$$

$= 31\frac{2}{3}$ *yds. the answer.*

2. How much in length that is $7\frac{7}{9}$ inches broad will make a foot square? *Ans.* 18 $\frac{18}{35}$ *inches.*

3. How much in length that is 11 $\frac{11}{12}$ poles broad will make a square acre? *Ans.* $\frac{61}{143}$ *po.*

4. If when wheat is 5 *s. per* bushel, the penny-loaf weighs 6 $\frac{9}{10}$ *oz.* what ought it to weigh when wheat is 8 *s.* 6 *d. per* bushel? *Ans.* 4 $\frac{1}{17}$ *oz.*

5. If when the days are 13 ⅝ hours long a traveller performs his journey in 35 ½ days; in how many days will he perform the same journey when the days are 11 $\frac{9}{10}$ hours long? *Ans.* 40 $\frac{615}{952}$ *days.*

6. How

6. How many yards of ell wide flannel are sufficient to line a cloak, containing 18 $\frac{7}{8}$ *yds.* of camblet $\frac{3}{4}$ yard wide? *Ans.* 11 *yds.* 1 *qr.* 1 $\frac{4}{5}$ *na.*

7. A regiment of soldiers consisting of 976 men are to be new clothed, each coat to contain 2 $\frac{1}{2}$ yards of cloth that is 1 $\frac{5}{8}$ *yd.* wide, and lined with shalloon $\frac{7}{8}$ *yd.* wide; how many yards of shalloon will line them? *Ans.* 4531 *yds.* 1 *qr.* 2 $\frac{6}{7}$ *na.*

8. If a coat and waistcoat can be made of 3 $\frac{3}{4}$ *yds.* of broad cloth of 1 $\frac{1}{2}$ *yds.* in breadth, how many yards of stuff of $\frac{5}{8}$ *yds.* in breadth will it require to fit the same person? *Ans.* 9 *yds.*

DECIMAL FRACTIONS.

A *decimal fraction* is that whose denominator is an unit with as many cyphers annexed as the numerator has places; and is usually expressed by writing the numerator only, with a point before it, on the left hand: thus, $\frac{5}{10}$, $\frac{25}{100}$, $\frac{75}{1000}$, $\frac{123}{100000}$, &c. are decimal fractions, and are expressed by .5 .25 .075 and .00123 respectively.

The 1*st.* 2*d.* 3*d.* 4*th*, &*c.* places of decimals, counting from the left hand towards the right, are called primes, seconds, thirds, fourths, &*c.*

Cyphers to the right hand of decimals make no alteration in their value; for .5 .50 .500, &*c.* are decimals, having the same value, being each = $\frac{1}{2}$; but if they are placed on the left hand they decrease their value in a ten-fold proportion. Thus, .5, .05, .005, &*c.* are 5 tenth parts, 5 hundredth parts, 5 thousandth parts, &*c.* respectively. *

* As in notation of whole numbers the value of the figures increase in a ten-fold proportion, from the right hand to the left; so, in decimals, their values decrease in the same ten-fold proportion, from the left hand to the right. Thus, .5 expresses 5 tenth parts of the integer, .05, 5 hundredth parts, &c.

ADDI-

ADDITION OF DECIMALS.

RULE.

1. Place the numbers under each other according to the value of their places.

2. Find their ſum as in whole numbers, and point off as many places, for decimals, as are equal to the greateſt number of decimal places in any of the given numbers.

EXAMPLES.

1. Find the ſum of 25.074 + 1.8254 + 125 + .0567876 + 1776.111.

```
  25.074
   1.8254
 125
    .0567876
1776.111
------------
1928.0671876 the sum.
```

2. Find the ſum of 376.25 + 86.125 + 637.4725 + 6.5 + 358.865 + 41.02. *Anſ.* 1506.2325.

3. Required the ſum of 3.5 + 47.25 + 927.01 + 2.0073 + 1.5. *Anſ.* 981.2673

4. Required the ſum of 276 + 54.321 + .65 + 112 + 12.5 + .0463. *Anſ.* 455.5173

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value; then ſubtract as in whole numbers, and point off the decimals as in addition.

EXAMPLES.

1. Find the difference of 2464.21 and 327.07643.

2464.21

2464.21
327.07643

2137.13357 *the difference.*

2. From 127.62 take 13.725. *Anſ.* 113.895
3. From 6213.725 take 162.25. *Anſ.* 6051.475
4. From 3760.279 take 423.0076. *Anſ.* 3337.2714

MULTIPLICATION OF DECIMALS.

RULE*

1. Place the factors and multiply them as in whole numbers.

2. Point off as many figures from the product as there are decimal places in both the factors; and if there are not ſo many places in the product, ſupply the defect by prefixing cyphers.

EXAMPLES.

Multiply .02534
by .03256

```
      15204
     12670
     5068
    7602
.0008250704 the product.
```

* To prove the truth of the rule, let .9776 and .823 be the numbers to be multiplied; now theſe are equivalent to $\frac{9776}{10000}$ and $\frac{823}{1000}$; whence $\frac{9776}{10000} \times \frac{823}{1000} = \frac{8045648}{10000000} =$ .8045648 by the nature of notation, and conſiſting of as many places as there are cyphers, that is, of as many places as are in both the numbers; and the ſame is true of any two numbers whatever.

2. Multiply 79.347 by 23.15. *Ans.* 1836.88305
3. Multiply .63478 by .8204. *Ans.* .524582192
4. Multiply .385746 by .00463.

Ans. 00178600398

CASE 2.

To contract the operation, so as to retain as many decimal places in the product as may be thought necessary.

RULE.

1. Write the units place of the multiplier under that figure of the multiplicand whose place you would reserve in the product; and dispose of the rest of the figures in a contrary order to what they are usually placed in.

2. In multiplying, reject all the figures that are to the right hand of the multiplying digit, and set down the products, so that their right hand figures may fall in a straight line below each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, &c. from the preceding figures when you begin to multiply, and the sum is the product required.

EXAMPLES.

1. It is required to multiply 73.8429753 by 4.628754 and to retain only five places of decimals in the product.

Contracted way.	*Common way.*	
73.8429753	73.	8429753
457826.4	4.	628754
29537190	29	53719012
4430579	369	2148765
147686	5169	008271
59074	59074	38024
5169	147685	9506
369	4430578	518
30	29537190	12
341.80097	341 80096	72917762

2. Multiply 245.378263 by 72.4385 reserving 5 places of decimals in the product. *Ans.* 17774.83330

3. Multiply .248264 by .725234 reserving 6 figures, 5 figures and 4 figures in the product respectively.

Ans. .180049. 18005, *and* .1800

4. Multiply 8634.875 by 843.7527 reserving only the integers in the product. *Ans.* 7285699

DIVISION OF DECIMALS.

RULE.*

1. Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient are not so many as the rule requires, supply the defect by prefixing cyphers.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be prefixed to the dividend, and the quotient carried on to any degree of exactness.

EXAMPLES.

```
179).48624097(.00271643.2685)27.0000(100.55865
     1282                          15000
      294                           15750
      1150                           23250
       769                            17700
       537                             15900
      ———                              24750
                                      ———
                                       &c.
```

* The reason of pointing off as many decimal places in the quotient as those in the dividend exceed the divisor, will easily appear; for since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of multiplication; it therefore follows, that the quotient contains as many as the dividend exceeds the divisor.

1. Divide

1. Divide 14 by .7854. *Anf.* 17.825 &c.
2. Divide 234.70525 by 64.25 *Anf.* 3.653
3. Divide 217.568 by 100. *Anf.* 2.17568
4. Divide .8727587 by .162 *Anf.* 5.38739 &c.

CASE 2.

To contract the operation, fo as to retain as many decimal places in the quotient as may be thought neceffary.

RULE.

1. Take as many of the left hand figures of the divifor as are equal to the required number of decimal places in the quotient, and find how many times they may be had in the firft figures of the dividend, as ufual.

2. Let each remainder be a new dividend; and for every fuch dividend, leave out one figure to the right hand of the divifor, remembering to carry for the increafe of the figures cut off, as in the fecond rule of multiplication.

3. The decimal places of the quotient may be pointed off, by obferving that the firft figure of the quotient muft poffefs the fame place with that figure of the dividend which ftands over the units place of the firft product.

EXAMPLES.

1. Divide 2508.928065051 by 92.41035, fo as to have 4 places of decimals in the quotient.

Contracted way.

```
92.41035)2508.928065051(27.1498
           660721
            13849
             4608
              912
               80
                6
```

G *Common*

Common way.

```
92.41035)2508.928065051(27.1498
           660721|06
            13848|615
             4607|5800
              911|16605
               79|472904
                5|544621
```

2. Divide 721.17562 by 2.257432, and let there be only 3 places of decimals in the quotient.

Ans. 319.467

3. Divide 12.169825 by 3.14159 and preserve 5 places of decimals in the quotient. *Ans.* 3.87377

4. Divide 87.076326 by 9.365407 and let there be 7 places of decimals in the quotient. *Ans.* 9.2976554

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal one.

RULE.*

Divide the numerator by the denominator, and the quotient will be the decimal required.

EXAM-

---

* Let the given vulgar fraction whose decimal expression is required be $\frac{7}{13}$. Now since every decimal fraction has 10, 100, or 1000, &c, for its denominator; and, if two fractions are equal, it will be, as the denominator of one is to its numerator, so is the denominator of the other to its numerator; therefore $13 : 7 :: 1000 \text{ \&c.} : \frac{7 \times 1000}{13}$ &c. $= \frac{70000}{13}$ &c. $= .53846$ the numerator of the decimal required; and is the same as by the rule.

The

EXAMPLES.

1. Reduce $\frac{5}{24}$ to a decimal.

$$4)5.000000$$
$$6)1.250000$$
$$.208333, \&c.$$

2. Required the equivalent decimal expressions for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. *Ans.* 25 .5 *and* .75
3. What is the decimal of $\frac{3}{8}$? *Ans.* .375
4 What is the decimal of $\frac{1}{25}$? *Ans.* .04
5. What is the decimal of $\frac{3}{192}$? *Ans.* 015625
6. Express $\frac{275}{3842}$ decimally. *Ans.* .071577 &c.

CASE 2.

To reduce numbers of different denominations to their equivalent decimal values.

The following method of throwing a vulgar fraction, whose denominator is a prime number, into a decimal consisting of a great number of figures, is given by *Mr. Colson* in page 162 of *Sir Isaac Newton's Fluxions*.

EXAMPLE.

Let $\frac{1}{29}$ be the fraction proposed.

Then, by dividing in the common way till the remainder becomes a single figure, we shall have $\frac{1}{29} = .03448\ \frac{8}{29}$ for the complete quotient, and this equation being multiplied by the numerator 8 will give $\frac{8}{29} = 27584\ \frac{64}{29}$, or rather $\frac{8}{29} = .27586\ \frac{6}{29}$; and if this be substituted instead of the fraction in the first equation it will make $\frac{1}{29} = .0344827586\ \frac{6}{29}$. Again, let this equation be multiplied by 6, and it will give $\frac{6}{29} = .2068965517\ \frac{7}{29}$; and then by substituting as before $\frac{1}{29} = 03448275862068965517\ \frac{7}{29}$; and so on as far as may be thought proper.

R U L E.*

1. Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number for a divisor as will bring it to the next superior name, and draw a line between them.

3. Begin with the highest, and write the quotient of each division, as decimal parts, on the right hand of the dividend next below it; and so on till they are all used, and the last quotient will be the decimal sought.

EXAMPLES.

1. Reduce 15*s.* 9*d.* $\frac{3}{4}$ to the decimal of a pound.

4	3.
12	9.75
20	15.8125

.790625 *the decimal required.*

2. Reduce 9*s.* to the decimal of a pound. *Ans.* .45

3. Reduce 19*s.* $5\frac{1}{2}$*d.* to the decimal of a pound. *Ans.* .972916

4. Reduce 10 *oz.* 18 *dwt.* 16 *grs.* to the decimal of a *lb.* troy. *Ans.* .911111 &c.

5. Reduce 2 *qrs.* 14 *lb.* to the decimal of a *cwt.* *Ans.* .625 &c.

6. Reduce 17 *yds.* 1 *fo.* 6 *in.* to the decimal of a mile. *Ans.* .00994318 &c.

* The reason of the rule may be explained from the first example: thus, three farthings is $\frac{3}{4}$ of a penny, which brought to a decimal is .75; consequently $9\frac{3}{4}d.$ may be expressed 9.75 *d.*; but 9.75 is $\frac{975}{100}$ of a penny $= \frac{975}{1200}$ of a shilling, which brought to a decimal is .8125; and, therefore $15s.\ 9\frac{3}{4}d.$ may be expressed 15.8125*s.* In like manner 15.8125*s.* is $\frac{158125}{10000}$ of a shilling $= \frac{158125}{200000}$ of a pound, =, by bringing it to a decimal, to .790625*l.* as by the rule.

7. Reduce

7. Reduce 3 *qrs.* 2 *na.* to the decimal of a yard.
Ans. .875

8. Reduce 1 *ro.* 14 *po.* to the decimal of an acre.
Ans. .3375

9. Reduce 1 *gall.* of wine to the decimal of a *hhd.*
Ans. .015873

10. Reduce 3 *bu.* 1 *pe.* to the decimal of a quarter.
Ans. .40625

11. Reduce 10 *we.* 2 *da.* to the decimal of a year.
Ans. .1972602 &c.

CASE 3.

To find the decimal of any number of ſhillings, pence and farthings by inſpection.

RULE.*

Write half the greateſt even number of ſhillings for the firſt decimal figure, and let the farthings in the

* The invention of the rule is as follows: As ſhillings are ſo many 20ths of a pound, half of them muſt be ſo many 10ths, and conſequently take the place of 10ths. in the decimal; but when they are odd their half will always conſiſt of 2 figures, the firſt of which will be half the even number, next leſs, and the ſecond a 5; and this confirms the rule as far as it reſpects ſhillings.

Again, farthings are ſo many 960ths. of a pound; and had it happened that 1000, inſtead of 960, had made a pound, it is plain any number of farthings would have made ſo many thouſandths, and might have taken their place in the decimal accordingly. But 960 increaſed by $\frac{1}{24}$ part of itſelf is = 1000; conſequently any number of farthings increaſed by their $\frac{1}{24}$ part will be an exact decimal expreſſion for them. Whence if the number of farthings be more than 12, a $\frac{1}{24}$ part is greater than $\frac{1}{2}$, and therefore 1 muſt be added; and when the number of farthings is more than 37, a $\frac{1}{24}$ part is greater than 1 *d.* $\frac{1}{2}$, for which 2 muſt be added; and thus the rule is ſhewn to be right.

given pence and farthings possess the second and third places; observing to increase the second place by 5, if the shillings are odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 37.

EXAMPLES.

1. Find the decimal of 15 *s.* 8 ½ *d.* by inspection.

7 . = ½ *of* 14 *s.*
5 . *for the odd shilling.*
34 = *farthings in* 8½ *d.*
1 *for the excess of* 12.

.785 = *decimal required.*

2. Find by inspection the decimal expressions of 16 *s.* 4½ *d.* and 13 *s.* 10½ *d.* *Ans.* .819 *and* .694

3. Value the following sums by inspection, and find their total, *viz.* 19 *s.* 11¼ *d.* + 6 *s.* 2 *d.* + 12 *s.* 8¾ *d.* + 1 *s.* 10¼ *d.* + ¾ *d.* + 1¼ *d.* *Ans.* 2.043 *the total.*

CASE 4.

To find the value of any given decimal in terms of the integer.

RULE.

1. Multiply the decimal by the number of parts in the next less denomination, and cut off as many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the parts in the next inferior denomination, and cut off for a remainder as before.

3. Proceed in this manner through all the parts of the integer, and the several denominations, standing on the left hand, make the answer.

EXAM-

EXAMPLES.

1. Find the value of .37623 of a pound.

```
        20
   -------
   7.52460
        12
   -------
   6.29520
         4
   -------
   1.18080 Ans. 7 s. 6¼ d.
```

2. What is the value of .625 shilling? *Ans.* 7½ *d.*

3. What is the value of .8322916 *l.*? *Ans.* 16 *s.* 7½ *d.*

4. What is the value of .6725 *cwt*? *Ans.* 2 *qrs.* 19 *lb.* 5 *oz.*

5. What is the value of .67 of a league? *Ans.* 2 *mi.* 0 *fur.* 3 *po.* 1 *yd.* 3 *in.* 1 *bar.*

6. What is the value of .61 of a tun of wine? *Ans.* 2 *hhds.* 27 *gall.* 2 *qr.* 1 *pt.*

7. What is the value of .461 of a chaldron of coals? *Ans.* 16 *bu.* 2 *pe.*

8. What is the value of .42857 of a month? *Ans.* 1 *we.* 4 *da.* 23 *ho.* 59 *min.* 56 *se.*

CASE 5.

To find the value of any decimal of a pound by inspection.

RULE.

Double the first figure, or place of tenths, for shillings, and if the second be 5, or more than 5, reckon another shilling; then call the figures in the second and third places, after 5 is deducted, so many farthings, abating 1 when they are above 12, and 2 when above 37, and the result is the answer.

EXAMPLES.

1. Find the value of .785 *l.* by inspection.

14 *s.* = *double of* 7.
1 *s.* *for the* 5 *in the place of tenths.*
$8\frac{3}{4}d.$ = 35 *farthings.*
$\frac{1}{4}$ *for the excess of* 12, *abated.*

---

15 *s.* $8\frac{1}{2}$*d.* *the answer.*

2. Find the value of .875 *l.* by inspection.

Ans. 17 *s* 6 *d.*

3. Value the following decimals by inspection, and find their sum, *viz.* .927*l.* + .351*l.* + .203*l.* + .061*l.* + .020*l.* + .009*l.* *Ans.* 1 *l.* 11 *s.* $5\frac{3}{4}$ *d.*

RULE OF THREE IN DECIMALS.

EXAMPLES.

1. If $\frac{3}{8}$ of a yard cost $\frac{2}{5}$ of a pound, what will $\frac{5}{16}$ of an English ell cost?

$\frac{3}{8}$ = .375
$\frac{2}{5}$ = .4 *l.*
$\frac{1}{4}$ *ell* = $\frac{5}{16}$ *yd.* = .3125

.375 *yd.* : .4 *l.* :: .3125 *yd.*
.3125

.375).12500(.333 &c. = 6*s.* 8*d.* *the answer.*
1125

1250
1125

1250
1125

125

2. If

2. If an *oz.* of silver cost 5 *s.* 6 *d.* what is the price of a tankard that weighs 1 *lb.* 10 *oz.* 10 *dwts.* 4 *grs?*

Ans. 6*l.* 3*s.* 9½*d.*

3. If I buy 14 yards of cloth for 10 guineas, how many ells Flemish can I buy for 283*l.* 17*s.* 6*d.* at the same rate?

Ans. 504 *ells* 2 *qrs.*

4. How many *Eng. ells* of Holland may be bought for 25*l.* 18*s.* 1¾*d.* at 7*s.* 9½*d. per* yard?

Ans. 53 *Eng. ells* 1 *qr.*

CIRCULATING DECIMALS.

Circulating Decimals are produced from vulgar fractions whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. The circulating figures are called *repetends*; and if one figure only repeats, it is called a *single repetend*; as .1111, *&c*; .3333, *&c.*

2. A *compound repetend* hath the same figures circulating alternately; as .010101 *&c*; .123123123 *&c.*

3. If other figures arise before those that circulate, the decimal is called a *mixed repetend*; thus, .283333 *&c.* is a *mixed single repetend*, and .573 21321 *&c.* a *mixed compound repetend.*

4. A single repetend is expressed by writing only the circulating figure with a point over it: thus, .1111 *&c.* is denoted by $.\dot{1}$, and .333 *&c.* by $.\dot{3}$.

5. Compound repetends are distinguished by putting a point over the first and last repeating figure; thus, .0101 *&c.* is written $.\dot{0}\dot{1}$, and .123123 *&c.* $.\dot{1}2\dot{3}$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after the decimal point; thus, $.\dot{2}$ and $.\dot{3}$ are similar circulates; as are also $2.\dot{3}\dot{4}$ and $3.\dot{7}\dot{6}$ *&c.*

 6. *Dissimilar*

7. *Dissimilar repetends* consist of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $56.7\dot{8}98\dot{4}$, $8.5\dot{2}68\dot{3}$ and $.0\dot{5}67\dot{8}$, *&c.*

REDUCTION OF CIRCULATING DECIMALS.

CASE I.

To reduce a simple repetend to its equivalent vulgar fraction.

RULE.*

1. Make the given decimal the numerator, and let the denominator be a number consisting of as many nines as there are recurring places in the repetend.

2. If there are integral figures in the circulate, as many cyphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

---

* If unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; *i. e.* if $\frac{1}{9}$ be reduced to a decimal it will produce the circulate $.\dot{1}$; and since $.\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $.\dot{2}$ will $= \frac{2}{9}$, $\dot{3} = \frac{3}{9}$, and so on till $\dot{9} = \frac{9}{9} = 1$.

Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9.

Again, $\frac{1}{99}$, or $\frac{1}{999}$, being reduced to decimals, make $.010101$ *&c.* and $.001001$ *&c. ad infinitum*, $= .\dot{0}\dot{1}$ and $.\dot{0}0\dot{1}$; that is $\frac{1}{99} = .\dot{0}\dot{1}$ and $\frac{1}{999} = .\dot{0}0\dot{1}$; consequently $\frac{2}{99} = .\dot{0}\dot{2}$, $\frac{3}{99} = .\dot{0}\dot{3}$ *&c.*; and $\frac{2}{999} = .\dot{0}0\dot{2}$, $\frac{3}{999} = .\dot{0}0\dot{3}$ *&c.* and the same will hold universally.

EXAM-

EXAMPLES.

1. Required the least vulgar fractions equal to $.\dot{6}$ and $.\dot{1}2\dot{3}$. *Ans.* $.\dot{6} = \frac{6}{9} = \frac{2}{3}$; *and* $.\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}$.

2. Reduce $.\dot{3}$ to its equivalent vulgar fraction. *Ans.* $\frac{1}{3}$

3. Reduce $\dot{1}.6\dot{2}$ to its equivalent vulgar fraction. *Ans.* $\frac{1620}{999}$

4. Required the least vulgar fraction equal to $.\dot{7}6923\dot{0}$. *Ans.* $\frac{10}{13}$.

CASE 2.

To reduce a mixed repetend to its equivalent vulgar fraction.

RULE.*

1. To as many nines as there are figures in the repetend, annex as many cyphers as there are finite places, for a denominator.

2. Multiply the nines in the said denominator by the finite part, and add the repeating decimal to the product for the numerator.

3. If the repetend begins in some integral place, the finite value of the circulating part must be added to the finite part.

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* In like manner for a mixed circulate; consider it as divisible into its finite and circulating parts, and the same principle will be seen to run through them also; thus, the mixed circulate $.1\dot{6}$ is divisible into the finite decimal $.1$ and the repetend $.0\dot{6}$ but $.1 = \frac{1}{10}$ and $.0\dot{6}$ would be $= \frac{6}{9}$ provided the circulation began immediately after the place of units; but as it begins after the place of tens, it is $\frac{6}{9}$ of $\frac{1}{10} = \frac{6}{90}$, and so the vulgar fraction $= .1\dot{6}$ is $\frac{1}{10} + \frac{6}{90} = \frac{9}{90} + \frac{6}{90} = \frac{15}{90}$, and is the same as by the rule.

EXAMPLES.

1. What is the vulgar fraction equivalent to $.13\dot{8}$?

$9 \times 13 + 8 = 125 =$ *numerator, and* 900 *the denominator* $\therefore .13\dot{8} = \frac{125}{900} = \frac{5}{36}$ *the answer.*

2. What is the least vulgar fraction equivalent to $.5\dot{3}$?

Anf. $\frac{8}{15}$

3. What is the least vulgar fraction equal to $.5\dot{9}2\dot{5}$?

Anf. $\frac{16}{27}$

4. What is the least vulgar fraction equal to $.008\dot{4}9713\dot{3}$?

Anf. $\frac{83}{9768}$

CASE 3.

To make any number of diſſimilar repetends ſimilar and conterminous.

RULE.*

Change them into other repetends, which ſhall each conſiſt of as many figures as the leaſt common multiple of the ſeveral numbers of places, found in all the repetends, contains units.

EXAMPLES.

Diſſimilar		*Made ſimilar and conterminous.*
$9.\dot{8}1\dot{4}$	=	$9.81\dot{4}8148\dot{1}$
1.5	=	$1.50\dot{0}0000\dot{0}$
$87.2\dot{6}$	=	$87.26\dot{6}6666\dot{6}$
$.08\dot{3}$	=	$.08\dot{3}3333\dot{3}$
$124.\dot{0}\dot{9}$	=	$124.09\dot{0}909\dot{9}$

* Any given repetend whatever, whether ſingle, compound, pure or mixed, may be transformed into another repetend, that ſhall conſiſt of an equal, or greater number of figures at pleaſure: thus $.\dot{4}$ may be transformed to $.\dot{4}\dot{4}$, or $.\dot{4}4\dot{4}$, or $.4\dot{4}$, &c. Alſo $.\dot{5}\dot{7} = .\dot{5}75\dot{7} = .57\dot{5}\dot{7} = .5\dot{7}\dot{5}$; and ſo on; which is too evident to need any farther demonſtration.

2, Make

2. Make $.\dot{3}$ $.2\dot{7}$ and $.04\dot{5}$ ſimilar and conterminous.

3. Make $.\dot{3}2\dot{1}$, $.\dot{8}26\dot{2}$; $.\dot{0}\dot{5}$ and $.0\dot{9}0\dot{2}$ ſimilar and conterminous.

4. Make $.521\dot{7}$, $3.\dot{6}4\dot{3}$ and $17.\dot{1}2\dot{3}$ ſimilar and conterminous.

CASE 4.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will conſiſt of.

RULE.*

1. Reduce the given fraction to its leaſt terms, and divide the denominator by 2, 5 or 10 as often as poſſible.

2. Divide 9999 &c. by the former reſult till nothing remains, and the number of 9's uſed will ſhew the number of places in the repetend; which will begin

* In dividing 1.0000 &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as ſoon as the remainder is 1. And ſince 9999 &c. is leſs than 10000 &c. by 1, therefore 9999 &c. divided by any number whatever will leave 0 for a remainder, when the repeating figures are at their period. Now whatever number of repeating figures we have when the dividend is 1, there will be exactly the ſame number when the dividend is any other number whatever. For the product of any circulating number, by any other given number, will conſiſt of the ſame number of repeating figures as before. Thus, let .507650765076 &c. be a circulate whoſe repeating part is 5076. Now every repetend (5076) being equally multiplied, muſt produce the ſame product. For though theſe products will conſiſt of more places, yet the overplus in each, being alike, will be carried to the next, by which means each product will be equally increaſed, and conſequently every four places will continue alike. And the ſame will hold for any other number whatever.

Now from hence it appears, that the dividend may be altered at pleaſure, and the number of places in the repetend will ſtill be the ſame; thus, $\frac{1}{11} = .\dot{9}\dot{0}$, and $\frac{3}{11}$ or $\frac{1}{11} \times 3 = .\dot{2}\dot{7}$ where the number of places in each are alike, and the ſame will be true in all caſes.

after

after as many places of figures as there were 10's, 2's or 5's divided by.

If the whole denominator vanishes in dividing by 2, 5 or 10, the decimal will be finite, and will consist of as many places as you perform divisions.

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{210}{1120}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

$$\textit{First}\ 10\Big)\frac{210}{1120} = \frac{21}{112},\quad 2)112 = \overset{(2)}{56} = \overset{(2)}{28} = \overset{(2)}{14} = 7$$

Then $7\overline{)999999}$; *and therefore the decimal is infinite,*
$\quad\ 142857$

and the circulate consists of 6 places, beginning at the decimal point.

2. Let $\frac{1}{11}$ be the fraction proposed.
3. Let $\frac{2}{7}$ be the fraction proposed.
4. Let $\frac{13}{404}$ be the fraction proposed.
5. Let $\frac{1}{8544}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE.*

1. Make the repetends similar and conterminous, and find their sum as in common addition.

2. Divide this sum by as many nines as there are places in the repetend, and the remainder is the repetend of the sum; which must be set under the figures added, with cyphers on the left hand when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest as in finite decimals.

* These rules are both evident from what has been said in reduction.

EXAMPLES.

1. Let $3.\dot{6} + 78.3\dot{4}7\dot{6} + 735.\dot{3} + 375 + .\dot{2}\dot{7} + 187.\dot{4}$ be added together.

Dissimilar		*Sim. and Conterminous.*
$3.\dot{6}$	=	$3.6\dot{6}6666\dot{6}$
$78.3\dot{4}7\dot{6}$	=	$78.3\dot{4}7647\dot{6}$
$735.\dot{3}$	=	$735.3\dot{3}3333\dot{3}$
$375.$	=	$375.0\dot{0}0000\dot{0}$
$.\dot{2}\dot{7}$	=	$0.2\dot{7}2727\dot{2}$
$187.\dot{4}$	=	$187.4\dot{4}4444\dot{4}$
		$1380.0\dot{6}4819\dot{3}$ *the Product.*

In this question the sum of the repetends is 2648191, *which divided by* 999999 *gives* 2, *to carry, and the remainder* 648193.

2. Let $5391.357 + 72.3\dot{8} + 187.2\dot{1} + 4.296\dot{5} + 217.84\dot{9}\dot{6} + 42.17\dot{6} + .52\dot{3} + 58.3004\dot{8}$ be added together. *Ans.* $5974.1037\dot{1}$

3. Add $9.\dot{8}1\dot{4} + 1.5 + 87.2\dot{6} + 0.8\dot{3} + 124.0\dot{9}$ together. *Ans.* $222.75\dot{5}7239\dot{0}$

4. Add $\dot{1}6\dot{2} + 134.\dot{0}\dot{9} + 2.\dot{9}\dot{3} + 97.2\dot{6} + 3.\dot{7}6923\dot{0} + 99.08\dot{3} + 1.5 + .\dot{8}1\dot{4}$ together. *Ans.* $501.62\dot{6}51077\dot{7}$

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.

Make the repetends similar and conterminous, and subtract as usual; observing, that if the repetend of the number to be subtracted, be greater than the repetend of the number it is to be taken from, then the right-hand

hand figure of the remainder must be less by unity than it would be if the expressions were finite.

EXAMPLES.

1. From $85.\dot{6}\dot{2}$ take $13.7\dot{6}43\dot{2}$.

$$85.\dot{6}\dot{2} \quad = 85.6\dot{2}62\dot{6}$$
$$13.7\dot{6}43\dot{2} = \underline{13.7\dot{6}43\dot{2}}$$
$$71.8\dot{6}19\dot{3} \text{ \textit{the difference.}}$$

2. From $476.\dot{3}\dot{2}$ take $84.7\dot{6}9\dot{7}$. *Ans.* $391.552\dot{4}$

3. From $3.8\dot{5}\dot{6}4$ take $.0\dot{3}8\dot{2}$ *Ans.* $3.8\dot{1}$

MULTIPLICATION OF CIRCULATING DECIMALS.

RULE.

1. Turn both the terms into their equivalent vulgar fractions, and find the product of those fractions as usual.

2. Turn the vulgar fraction, expressing the product, into an equivalent decimal one, and it will be the product required.

EXAMPLES.

Multiply $.\dot{3}\dot{6}$ by $.2\dot{5}$.

$$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$$
$$.2\dot{5} = \frac{23}{90}$$

$$\frac{4}{11} \times \frac{23}{90} = \frac{92}{990} = .0\dot{9}2\dot{9} \text{ \textit{the product.}}$$

2. Multiply $37.2\dot{3}$ by $.2\dot{6}$. *Ans.* $9.92\dot{8}$

3. Multiply

3. Multiply $8574.\dot{3}$ by $87.\dot{5}$. *Ans.* $750730.\dot{5}1\dot{8}$

4. Multiply $3.\dot{9}7\dot{3}$ by 8. *Ans.* $31.\dot{7}9\dot{1}$

5. Multiply 49640.54 by $.\dot{7}050\dot{3}$.

Ans. $34998.4\dot{1}9900\dot{3}$

6. Multiply $3.\dot{1}4\dot{5}$ by $4.\dot{2}9\dot{7}$ *Ans.* $13.\dot{5}16953\dot{3}$

DIVISION of CIRCULATING DECIMALS.

RULE.

1. Change both the divisor and dividend into their equivalent vulgar fractions, and find their quotient as usual.

2. Turn the vulgar fraction, expressing the quotient, into its equivalent decimal, and it will be the quotient required.

EXAMPLES.

1. Divide $.\dot{3}\dot{6}$ by $.2\dot{5}$

$$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$$

$$.2\dot{5} = \frac{23}{90}$$

$$\frac{4}{11} \div \frac{23}{90} = \frac{4}{11} \times \frac{90}{23} = \frac{360}{253} = 1\tfrac{107}{253} \textit{ the quotient.}$$

2. Divide $319.28007\dot{1}1\dot{2}$ by 764.5. *Ans.* $.4176\dot{3}2\dot{5}$

3. Divide $234.\dot{6}$ by $.\dot{7}$. *Ans.* $301.\dot{7}1428\dot{5}$

4. Divide $13.\dot{5}16953\dot{3}$ by $4.29\dot{7}$. *Ans.* $3.\dot{1}4\dot{5}$

DUODECIMALS.

Duodecimals, or *Cross Multiplication*, is a rule made use of by workmen and artificers in casting up the contents of their works.

Dimensions

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus: primes ($'$), seconds ($''$), thirds, ($'''$), fourths (^{iv}), &c.

Artificers work is computed by different measures, *viz.*

1. Glazing, and mason's flatwork by the foot.

2. Painting, paving, plaistering, &c. by the yard.

3. Partioning, flooring, roofing, tiling, &c. by the square of 100 feet.

4. Brickwork, &c. by the rod of $16\frac{1}{2}$ feet, whose square is $272\frac{1}{4}$.

Note. Bricklayers always value their work at the rate of a brick and a half thick; and if the wall is more or less, it must be reduced to that thickness.

RULE.

1. Under the multiplicand, write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.

3. In the same manner, multiply all the multiplicand by the primes in the multiplier, and set the result of each term one place removed to the right-hand of those in the multiplicand.

4. Do the same with the seconds in the multiplier, setting the result of each term two places removed to the right hand of those in the multiplicand.

5. Proceed in like manner with all the rest of the denominations, and their sum will give the answer required.

EXAMPLES.

EXAMPLES.

1. Multiply 10*fe.* : 4′ : 5″ by 7*fe.* : 8′ : 6″

10*fe*:	4′ :	5″		
7 :	8 :	6		
72 :	6 :	11		
6 :	10 :	11 :	4	
	5 :	2 :	2 :	6
79*fe*:	11′ :	0″ :	6‴ :	6^{iv} *answer.*

2. Multiply 4*fe.* : 6′ by 14*fe.* : 9′.

Anſ. 66 *fe.* : 4′ : 6″

3. What is the product of 39*fe.* : 10′ : 7″ by 18 *fe.* 8′ : 4″? *Anſ.* 745*fe.* : 6′ : 10″ : 2‴ : 4^{iv}.

4. Multiply 24*fe.* : 10′ : 8″ : 7‴ : 5^{iv} by 9*fe.* : 4′ : 6″.

Anſ. 233*fe.* : 4′ : 5″ : 9‴ : 6^{iv} : 4^{v} : 6^{vi}.

5. Multiply 368*fe* : 7′ : 5″ by 137*fe.* : 8′ : 4″.

Anſ. 50756*fe.* : 7′ : 10″ : 9‴ : 8^{iv}

6. What is the price of a marble ſlab, whoſe length is 5*fe.* : 7′ and breadth 1*fo.* : 10′, at 6*s. per* foot.

Anſ. 3*l.* 1*s.* 5*d.*

7. There is a houſe with 3 tier of windows, 3 in a tier, the height of the firſt tier is 7*fe.* : 10′, of the ſecond 6*fe.* : 8′, and of the third 5*fe.* 4′, and the breadth of each is 3*fe* : 11′ : what will the glazing come to at 14*d. per* foot? *Anſ.* 13*l.* 11*s.* 10 *d.*

8. A room is to be ceiled, whoſe length is 74*fe.* : 9′ and width 11*fe.* : 6′ : what will it come to at 3*s.* 10½*d. per* yard? *Anſ.* 18*l.* 10*s.* 1*d.*

9. What will the paving a court yard come to at 4¾*d. per* yard, the length being 58 *fe.* : 6′ and breadth 54 *fe.* 9′? *Anſ.* 7*l.* 0*s.* 10*d.*

10. A

10. A room is 97 *fe.* : 8′ about, and 9 *fe.* : 10′ high: what will the painting of it come to, at 2 *s.* $8\frac{3}{4}$ *per* yard? *Anf.* 14 *l.* 11 *s.* $2\frac{1}{2}$*d.*

11. A piece of wainfcotting is 8 *fe.* : 3′ long, and 6 *fe.* : 6′ broad: what will it come to at 6 *s.* $7\frac{1}{2}$*d.* *per* yard? *Anf.* 1 *l.* 19 *s.* 5 *d.*

12. If a houfe meafures within the walls 52 *fe.* : 8′ in length, and 39 *fe.* : 6′ in breadth, and the roof be of a true pitch, or the rafters $\frac{3}{4}$ of the breadth of the building, what will it come to roofing at 10 *s.* 6 *d.* *per* fquare? *Anf.* 12 *l.* 12 *s.* $11\frac{3}{4}$*d.*

13. What will the tyling of a barn coft at 25 *s.* 6 *d.* *per* fquare, the length being 43 *fe.* : 10′ and the breadth 27 *fe.* : 5′ on the flat, the eave boards projecting 16 inches on each fide? *Anf.* 24 *l.* 9 *s.* $5\frac{1}{2}$*d.*

14. How many fquare rods are there in a wall $62\frac{1}{2}$ feet long, 14 *fe.* : 8′ high, and $2\frac{1}{2}$ bricks thick?

Anf. 5 *rods* 167 *fe.*

15. If a garden wall be 254 feet round, and 12 *fe.* : 7′ high, and 3 bricks thick, how many rods doth it contain? *Anf.* 23 *rods*, 136 *fe.*

SIMPLE INTEREST BY DECIMALS.

RULE.*

Multiply the principal, ratio, and time together, and it will give the intereft required.

---

* The following theorems will fhew all the poffible cafes of fimple intereft, where p = principal, t = time, r = ratio, and a = amount.

I. $ptr + p = a.$ II. $\frac{a - p}{rp} =$

III. $\frac{a}{tr + 1} = p.$ IV. $\frac{a - p}{tp} = r.$

Ratio

Ratio is the simple interest of 1 *l.* for 1 year, at the rate *per cent.* agreed on; thus the ratio

$$\text{at} \begin{cases} 3 & \text{---} & .03 \\ 3\frac{1}{2} & \text{---} & .035 \\ 4 & \textit{per cent. is} & .04 \\ 4\frac{1}{2} & \text{---} & .045 \\ 5 & \text{---} & .05 \end{cases}$$

EXAMPLES.

1. What is the interest of 945 *l.* 10 *s.* for 3 years, at 5 *per cent. per ann?*

$$\begin{array}{r} 945.5 \\ .05 \\ \hline 47.275 \\ 3 \\ \hline 141.825 \\ 20 \\ \hline 16.500 \\ 12 \\ \hline 6.000 \end{array}$$

Ans. 141 *l.* 16 *s.* 6 *d.*

2. What is the interest of 796 *l.* 15 *s.* for 5 years, at 4 *per cent per ann?* *Ans.* 179 *l.* 5 *s.* $4\frac{1}{2}$ *d.*

3. What is the simple interest of 880 *l.* for $1\frac{1}{4}$ years, at $3\frac{1}{2}$ *per cent. per ann?* *Ans.* 38 *l.* 10 *s.*

4. What is the interest of 537 *l.* 15 *s.* from November 11th, 1764, to June 5th, 1765, at $3\frac{5}{8}$ *per cent?* *Ans.* 11 *l.* 0 *s.* $\frac{1}{4}$ *d.*

DISCOUNT

10. A room is 97 *fe.* : 8′ about, and 9 *fe.* : 10′ high: what will the painting of it come to, at 2 *s.* $8\frac{3}{4}$ *per* yard? *Anſ.* 14 *l.* 11 *s.* $2\frac{1}{2}$ *d.*

11. A piece of wainſcotting is 8 *fe.* : 3′ long, and 6 *fe.* : 6′ broad: what will it come to at 6 *s.* $7\frac{1}{2}$ *d.* *per* yard? *Anſ.* 1 *l.* 19 *s.* 5 *d.*

12. If a houſe meaſures within the walls 52 *fe.* : 8′ in length, and 39 *fe.* : 6′ in breadth, and the roof be of a true pitch, or the rafters $\frac{3}{4}$ of the breadth of the building, what will it come to roofing at 10 *s.* 6 *d.* *per* ſquare? *Anſ.* 12 *l.* 12 *s.* $11\frac{3}{4}$ *d.*

13. What will the tyling of a barn coſt at 25 *s.* 6 *d.* *per* ſquare, the length being 43 *fe.* : 10′ and the breadth 27 *fe.* : 5′ on the flat, the eave boards projecting 16 inches on each ſide? *Anſ.* 24 *l.* 9 *s.* $5\frac{1}{2}$ *d.*

14. How many ſquare rods are there in a wall $62\frac{1}{2}$ feet long, 14 *fe.* : 8′ high, and $2\frac{1}{2}$ bricks thick?

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SIMPLE INTEREST BY DECIMALS.

RULE.*

Multiply the principal, ratio, and time together, and it will give the intereſt required.

---

* The following theorems will ſhew all the poſſible caſes of ſimple intereſt, where p = principal, t = time, r = ratio, and a = amount.

I. $ptr + p = a$. II. $\frac{a - p}{rp} =$

III. $\frac{a}{tr + 1} = p$. IV. $\frac{a-p}{tp} = r$.

Ratio

Ratio is the ſimple intereſt of 1 *l.* for 1 year, at the rate *per cent.* agreed on; thus the ratio

at	3	——	.03
	$3\frac{1}{2}$	——	.035
	4	*per cent.* is	.04
	$4\frac{1}{2}$	——	.045
	5	——	.05

EXAMPLES.

1. What is the intereſt of 945 *l.* 10 *s.* for 3 years, at 5 *per cent. per ann?*

```
 945.5
   .05
 ------
 47.275
      3
 ------
 141.825
     20
 ------
 16.500
     12
 ------
  6.000
```

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3. What is the ſimple intereſt of 880 *l.* for $1\frac{1}{4}$ years, at $3\frac{1}{2}$ *per cent. per ann?* *Anſ.* 38 *l.* 10 *s.*

4. What is the intereſt of 537 *l.* 15 *s.* from November 11th, 1764, to June 5th, 1765, at $3\frac{5}{8}$ *per cent?* *Anſ.* 11 *l.* 0 *s.* $\frac{1}{4}$*d.*

DISCOUNT

DISCOUNT BY DECIMALS.

RULE.*

As the amount of 1 *l.* for the given time, is to 1 *l.* so is the intereſt of the debt for the ſaid time, to the diſcount required.

Subtract the diſcount from the principal, and the remainder will be the preſent worth.

EXAMPLES.

1. What is the diſcount of 573 *l.* 15 *s.* due 3 years hence, at $4\frac{1}{2}$ *per cent. per annum?*

$.045 \times 3 + 1 = 1.135$ = *amount of* 1 *l. for the given time.*

And $573.75 \times .045 \times 3 = 77.45625$ = *intereſt of the debt for the given time.*

1.135 : 1 :: 77.45625

1.135)77.45625(68.243
6810
9356
9080
2762
2270
4925
4540
3850
3405
445

68.243 = 68 *l.* 4 *s.* $10\frac{1}{4}$*d. the anſwer.*

---

* Let *m* repreſent any debt, and *n* the time of payment; then will the following tables exhibit all the variety that can happen with reſpect to preſent worth and diſcount.

2. What

2. What is the discount of 725 *l.* 16 *s.* for 5 months at $3\frac{7}{8}$ *per cent. per annum?* *Ans.* 11 *l.* 10 *s.* $3\frac{1}{4}$*d.*

3. What ready money will discharge a debt of 1377 *l.* 13 *s.* 4 *d.* due 2 years, 3 quarters and 25 days hence, discounting at $4\frac{3}{8}$ *per cent. per annum?*

Ans. 1226 *l.* 8 *s.* $8\frac{1}{2}$*d.*

EQUATION OF PAYMENTS BY DECIMALS.

Having two debts due at different times, to find the equated time to pay the whole at once.

Of the present worth of money paid before it is due at simple interest.			
The present worth of any sum *m*.			
Rate *per cent.*	For *n* years	*n* months	*n* days
r *per cent.*	$\frac{100m}{nr + 100}$	$\frac{200m}{nr + 1200}$	$\frac{36500m}{nr + 36500}$
3 *per cent.*	$\frac{100m}{3n + 100}$	$\frac{400m}{n + 400}$	$\frac{36500m}{3n + 36500}$
4 *per cent.*	$\frac{25m}{n + 25}$	$\frac{300m}{n + 300}$	$\frac{9125m}{n + 9125}$
5 *per cent.*	$\frac{20m}{n + 20}$	$\frac{240m}{n + 240}$	$\frac{7300m}{n + 7300}$

R U L E.*

1. To the sum of both payments, add the continual product of the first payment, the rate, or interest of 1*l.* for 1 year, and the time between the payments, and call this the first number.

2. Multiply

Of discounts to be allowed for paying of money before it falls due at simple interest.

The discount of any sum m.

Rate *per cent.*	For n years	n months	n days
r *per cent.*	$\frac{mnr}{nr+100}$	$\frac{mnr}{nr+1200}$	$\frac{mnr}{nr+36500}$
3 *per cent.*	$\frac{3mn}{3n+100}$	$\frac{mn}{n+400}$	$\frac{3mn}{3n+36500}$
4 *per cent.*	$\frac{mn}{n+25}$	$\frac{mn}{n+30}$	$\frac{mn}{n+9125}$
5 *per cent.*	$\frac{mn}{n+20}$	$\frac{mn}{n+240}$	$\frac{mn}{n+7300}$

* No rule in arithmetic has been the occasion of so many disputes as that of Equation of Payments. Almost every writer upon this subject has endeavoured to shew the fallacy of the methods made use of by other authors, and to substitute a new one in their stead. But the only true rule, as it appears to me, is that given by

2. Multiply twice the first payment by the rate, and call this the second number.

3. Divide the first number by the second, and call the quotient the third number.

4. Call

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by *Mr. Malcolm in page 621 of his Arithmetic,* the principles of which are derived from the consideration of interest and discount.

The rule, given above, is the same as *Mr. Malcom's*, except that it is not incumbered with the time before any payment is due, that being no necessary part of the operation.

Demon. of the Rule. Suppose a sum of money to be due immediately, and another sum at the expiration of a certain given time forward, and it is proposed to find a time to pay the whole at once, so that neither party shall sustain loss.

Now, it is plain, that the equated time must fall between the two payments; and that what is got by keeping the first debt after it is due, should be equal to what is lost by paying the second debt before it is due.

But the gain arising from the keeping of a sum of money, after it is due is, evidently, equal to the interest of the debt for that time.

And the loss which is sustained by the paying of a sum of money before it is due is, evidently, equal to the discount of the debt for that time.

Therefore, it is obvious, that the debtor must retain the sum immediately due, or the first payment, till its interest shall be equal to the discount of the second sum for the time it is paid before due; because, in that case, the gain and loss will be equal, and consequently neither party can be the loser.

Now, to find such a time, let $a =$ 1st. payment, $b =$ second, and $t =$ time between the payments; $r =$ rate, or interest of 1 *l.* for 1 year, and $x =$ equated time after the first payment.

Then $arx =$ interest of a for x time.

and $\frac{btr-brx}{1+tr-rx} =$ discount of b for the time $t-x$.

But $arx = \frac{btr-brx}{1+tr-rx}$ by the question, from which equation x is found $= \frac{a+b+atr}{2ar} \pm \left|\overline{\frac{a+b+atr}{2ar}}\right|^2 - \frac{bt}{ar}\Big|^{\frac{1}{2}}$

Let $\frac{a+b+atr}{2ar}$ be put equal to n, and $\frac{bt}{ar} = m$.

4. Call the square of the third number the fourth number.

5. Divide

Then it is evident that n, or its equal $\overline{n^2}|^{\frac{1}{2}}$, is greater than $\overline{n^2 - m}|^{\frac{1}{2}}$, and therefore x will have two affirmative values, the quantities $n + \overline{n^2 - m}|^{\frac{1}{2}}$ and $n - \overline{n^2 - m}|^{\frac{1}{2}}$ being both positive.

But only one of those values will answer the conditions of the question; and, in all cases of this problem, x will be $= n - \overline{n^2 - m}|^{\frac{1}{2}}$.

For suppose the contrary, and let $x = n + \overline{n^2 - m}|^{\frac{1}{2}}$. Then $t - x = t - n - \overline{n^2 - m}|^{\frac{1}{2}} = \overline{\overline{t - n}|^2}|^{\frac{1}{2}} - \overline{n^2 - m}|^{\frac{1}{2}}$. $= \overline{t^2 - 2tn + n^2}|^{\frac{1}{2}} - \overline{n^2 - m}|^{\frac{1}{2}} = \overline{n^2 + t^2 - 2tn}|^{\frac{1}{2}} - \overline{n^2 - m}|^{\frac{1}{2}}$.

Now, since $\overline{a + b + atr} \times \frac{1}{2ar} = n$, and $bt \times \frac{1}{ar} = m$, we shall have, from the first of these equations, $t^2 - 2tn = \overline{-bt - at} \times \frac{1}{ar}$, and consequently $t - x = \overline{n^2 - \overline{bt - at} \times \frac{1}{ar}}|^{\frac{1}{2}} - \overline{n^2 - bt \times \frac{1}{ar}}|^{\frac{1}{2}}$.

But $\overline{n^2 - bt \times \frac{1}{ar}}|^{\frac{1}{2}}$ is evidently greater than $\overline{n^2 - \overline{bt - at} \times \frac{1}{ar}}|^{\frac{1}{2}}$, and therefore $\overline{n^2 - bt \times \frac{1}{ar}}|^{\frac{1}{2}} - \overline{n^2 - \overline{bt - at} \times \frac{1}{ar}}|^{\frac{1}{2}}$, or its equal $t - x$, must be a negative quantity; and consequently x will be greater than t, that is, the equated time will fall beyond the second payment, which is absurd. The value of x, therefore, cannot

5. Divide the product of the second payment, and time between the payments, by the product of the first payment and the rate, and call the quotient the fifth number.

6. From the fourth number take the fifth, and call the square root of the difference the sixth number.

7. Then the difference of the third and sixth numbers is the equated time, after the first payment is due.

EXAMPLES.

1. There is 100 *l.* payable 1 year hence, and 105 *l.* payable 3 years hence: what is the equated time, allowing simple interest at 5 *per cent. per annum?*

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not be $= \frac{a+b+atr}{2ar} + \left|\overline{\left.\frac{a+b+atr}{2ar}\right|^2 - \frac{bt}{ar}}\right|^{1}$, but

must in all cases be $= \frac{a+b+atr}{2ar} - \left|\overline{\left.\frac{a+b+atr}{2ar}\right|^2 - \frac{bt}{ar}}\right|^{\frac{1}{2}}$,

which is the same as the rule.

From this it appears, that the double sign made use of by Mr. *Malcolm*, and every author since, who has given his method, cannot obtain, and that there is no ambiguity in the problem.

In like manner it might be shewn, that the directions usually given for finding the equated time when there are more than two payments will not agree with the hypothesis, but this may be easily seen by working an example at large, and examining the truth of the conclusion.

The equated time for any number of payments may be readily found when the question is proposed in numbers, but it would not be easy to give algebraic theorems for those cases, on account of the variation of the debts and times, and the difficulty of finding between which of the payments the equated time would happen.

Supposing r to be the amount of 1 *l.* for 1 year, and the other letters as before, then $t - \frac{\log.\frac{ar^t+b}{s}}{\log.r}$ will be a general theorem for the equated time of any two payments, reckoning compound interest, and is found in the same manner as the former.

```
      100              100
      .05                2
     ----             ----
     5.00              200
        2              .05
     ----             ----
    10.00             10.00 = 2d. number.
    100
    105
   -----
 10)215 = 1st. number.
   -----
    21.5 = 3d. number.
    21.5
   -----
    1075
    215
   430
   -----
  462.25 = 4th. number.

                 105
                   2
                ----
1st. payment × rate = 5)210
                ----
                  42 = 5th. number.

    462.25
     42
    ------       21.5
    420.25      (20.5 = 6th. number.
      4          ----
    ------       1 = equated time from the
405)2025             first payment, & ∴ 2
    2025             years = whole equated
    ----             time.
```

2. Suppose

2. Suppose 400 *l.* is to be paid at the end of 2 years, and 2100 *l.* at the end of 8 years: what is the equated time for one payment, reckoning 5 *per cent.* simple interest? *Ans.* 7 *years.*

3. Suppose 300 *l.* is to be paid at one year's end, and 300 *l.* more at the end of 1 ½ years; it is required to find the time to pay it at one payment, allowing 5 *per cent.* simple interest. *Ans.* 1.248637 *years.*

COMPOUND INTEREST BY DECIMALS.

RULE.*

1. Find the amount of 1 *l.* for 1 year at the given rate *per cent.*

2. Involve the amount thus found to such a power as is denoted by the number of years.

3. Multiply this power by the principal, or given sum, and the product will be the amount required.

4. Subtract the principal from the amount, and the remainder will be the interest.

EXAM-

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* *Demon.* Let $r =$ amount of 1 *l.* for 1 year, and $p =$ principal or given sum; then, since r is the amount of 1 *l.* for 1 year, r^2 will be its amount for 2 years, r^3 for 3 years, and so on; for, when the rate and time is the same, all principal sums are necessarily as their amounts; and consequently as r is the principal for the second year, it will be as $1 : r :: r : r^2$ = amount for the second year, or principal for the third; and again, as $1 : r :: r^2 : r^3$ = amount for the third year, or principal for the fourth, and so on to any number of years. And if the number of years be denoted by t, the amount of 1 *l.* for t years will be r^t. From hence it will appear, that the amount of any other principal sum p for t years is pr^t; for as $1 : r^t :: p : pr^t$, the same as in the rule.

If the rate of interest is determined to any other time than a year, as $\frac{1}{2}$, $\frac{1}{4}$ &c. the rule is the same, and then t will represent that stated time.

Let

EXAMPLES.

1. What is the compound interest of 500 *l.* for 4 years at 5 *per cent. per annum?*

1.05

Let $\begin{cases} r = \text{amount of 1\textit{l}. for 1 year, at the given rate \textit{per cent}.} \\ p = \text{principal, or sum put out to interest,} \\ i = \text{interest,} \\ t = \text{time,} \\ m = \text{amount for the time } t, \end{cases}$

Then the following theorems will exhibit the solutions of all the cases in compound interest.

I. $pr^t = m$, II. $pr^t - p = i$,

III. $\dfrac{m}{r^t} = p$, IV. $\left.\dfrac{m}{p}\right|^{\frac{1}{t}} = r$,

The most convenient way of giving the theorem for the *time*, as well as for all the other cases, will be by logarithms, as follows:

I. $t \times \log. r + \log p = \log. m$, II. $\log. m - t \times \log. r = \log. p$.

III. $\dfrac{\log. m - \log. p}{\log. r} = t$, IV. $\dfrac{\log. m - \log. p}{t} = \log. r$.

If the compound interest, or amount of any sum, be required for the parts of a year, it may be determined as follows:

I. When the time is any aliquot part of a year.

RULE.

1. Find the amount of 1 *l.* for 1 year, as before, and that root of it which is denoted by the aliquot part, will be the amount sought.

2. Multiply the amount thus found by the principal, and it will be the amount of the given sum required.

II. When the time is not an aliquot part of a year.

RULE.

1. Reduce the time into days, and the 365th. root of the amount of 1 *l.* for 1 year, is the amount for 1 day.

2. Raise

$$
\begin{array}{r}
1.05 = \textit{amount of 1l. for 1 year at 5 per cent.} \\
1.05 \\
\hline
525 \\
1050 \\
\hline
1.1025 \\
1.1025 \\
\hline
55125 \\
22050 \\
110250 \\
11025 \\
\hline
1.21550625 = \textit{4th. power of 1.05.} \\
500 = \textit{principal.} \\
\hline
\end{array}
$$

———— = *amount.*

607.75312500
500

107.753125 = 107*l.* 15*s.* 0¾*d.* = *interest required.*

2. What is the amount of 760 *l.* 10 *s.* for 4 years at 4 *per cent?* *Ans.* 889 *l.* 13*s.* 6½*d.*

3. What is the compound interest of 760 *l.* 10 *s.* for 4 years, at 4 *per cent. per annum?* *Ans.* 129*l.* 3*s.* 6½*d.*

4. What is the amount of 721 *l.* for 21 years, at 4 *per cent. per annum?* *Ans.* 1642 *l.* 19 *s.* 10 *d.*

---

2. Raise this amount to that power whose index is equal to the number of days, and it will be the amount of 1*l.* for the given time.

3. Multiply this amount by the principal, and it will be the amount of the given sum required.

To avoid extracting very high roots the same may be done by logarithms, thus: divide the logarithm of the rate, or amount of 1*l.* for 1 year, by the denominator of the given aliquot part, and the quotient will be the logarithm of the root sought.

5. What is the amount of 217*l.* forborn $2\frac{1}{4}$ years, at 5 *per cent. per annum,* supposing the interest payable quarterly? *Ans.* 242*l.* 13*s.* $4\frac{1}{2}$*d.*

ANNUITIES.

An annuity is a sum of money payable every year for a certain number of years, or for ever.

When the debtor keeps the annuity in his own hands, beyond the time of payment, it is said to be in *arrears.*

The sum of all the annuities for the time they have been forborn, together with the interest due upon each, is called the *amount.*

If an annuity is to be bought off, or paid all at once, at the beginning of the first year, the price which ought to be given for it is called the *present worth.*

To find the Amount of an Annuity at Simple Interest.

RULE.*

1. Find the sum of the natural series of numbers 1, 2, 3, *&c.* to the number of years less one.

2. Multiply

---

* *Demon.* Whatever the time is, there is due upon the first year's annuity, as many year's interest as the whole number of years less one; and gradually one less upon every succeeding year to the last but one; upon which there is due only one year's interest, and none upon the last; therefore in the whole there is due as many year's interest of the annuity as the sum of the series, 1, 2, 3, 4 *&c.* to the number of years less one. Consequently one year's interest multiplied by this sum, must be the whole interest due; to which if all the annuities be added, the sum is plainly the amount. *Q. E. D.*

Let r be the ratio, n the annuity, t the time, and a the amount.

Then will the following theorems give the solutions of all the different cases.

I. $\dfrac{t^2rn - trn}{2} + tn = a,$ II. $\dfrac{2a - 2tn}{t^2n - tn} = r,$

III.

2. Multiply this ſum by one year's intereſt of the annuity, and the product will be the whole intereſt due upon the annuity.

3. To this product add the product of the annuity and time, and the ſum will be the amount ſought.

EXAMPLES.

1. What is the amount of an annuity of 50*l.* for 7 years, allowing ſimple intereſt at 5 *per cent*?

$1 + 2 + 3 + 4 + 5 + 6 = 21 = 3 \times 7$

£.	s.	
2	10	= 1 *year's intereſt of* 50 £.
	3	
7	10	
	7	
52	10	
350	0	= 50 *l.* × 7

402*l.* 10*s.* = *amount required.*

2. If a penſion of 600 *l.* *per ann.* be forborn 5 years, what will it amount to, allowing 4 *per cent.* ſimple intereſt? *Anſ.* 3240 *l.*

H 5 3. What

III. $\frac{2a}{t^2r - tr + 2t} = n$, IV. $\left|\frac{2a}{rn} + \frac{d}{4}\right|^{\frac{1}{2}} - \frac{d}{2} = t$.

In the laſt theorem $d = \frac{2n - rn}{rn}$, and in theorem 1ſt. if a ſum cannot be found equal to the amount, the problem is impoſſible in whole years.

Note, Some writers look upon this method of finding the amount of an annuity as a ſpecies of *compound intereſt*; the annuity itſelf, they ſay, being, properly, the ſimple intereſt, and the capital, from whence it ariſes, the principal.

3. What will an annuity of 250*l.* amount to in 7 years, to be paid by half yearly payments, at 6 *per cent. per annum*, simple interest? *Ans.* 2091 *l.* 5 *s.*

To find the present Worth of an Annuity at Simple Interest.

RULE.*

Find the present worth of each year by itself, discounting from the time it falls due, and the sum of all these will be the present worth required.

---

* The reason of this rule is manifest from the nature of discount, for all the annuities may be considered separately, as so many single and independent debts, due after 1, 2, 3 *&c.* years; so that the present worth of each being found, their sum must be the present worth of the whole.

This is *Kersey's* rule, as it is given in his appendix to *Wingate's* Arithmetic. Sir *Samuel Moreland*, *Ward*, &c. have represented it as very erroneous, and given another rule, which they say, brings out the true solution.

Now, granting the condition or agreement of allowing simple interest to be consistent, it appears to me that *Kersey's* rule is the true one, and the error which Sir *Samuel* and others complain of seems to lie all on their side.

But it would be needless to enter further into the merits of this dispute, since the purchasing of annuities by simple interest is in the highest degree unjust and absurd. One instance only will be sufficient to shew the truth of this assertion. The price of an annuity of 50*l.* to continue 40 years, discounting at 5 *per cent.* will, by either of the rules, amount to a sum of which one year's interest only exceeds the annuity. Would it not therefore be highly ridiculous to give, for an annuity to continue only 40 years, a sum which would yield a greater yearly interest for ever.

I have here shewn the method of computing annuities by simple interest, merely in compliance to custom; but would have it considered as a matter more of speculation than real use, it being not only customary, but also most equitable, to allow compound interest.

Let p = present worth, and the other letters as before.

$$\text{Then}\begin{cases} n \times \overline{\dfrac{1}{1+r} + \dfrac{1}{1+2r} + \dfrac{1}{1+3r} \text{ \&c. to } \dfrac{1}{1+tr}} = p \\[2ex] p \div \overline{\dfrac{1}{1+r} + \dfrac{1}{1+2r} + \dfrac{1}{1+3r} \text{ \&c. to } \dfrac{1}{1+tr}} = n. \end{cases}$$

The

EXAMPLES.

1. What is the present worth of an annuity of 100 *l.* to continue 5 years, at 6 *per cent. per ann.* simple interest?

106 : 100 : : 100 : 94.3396 = *present worth for* 1 *year.*
112 : 100 : : 100 : 89.2857 = ... 2*d. year.*
118 : 100 : : 100 : 84.7457 = ... 3*d. year.*
124 : 100 : : 100 : 80.6451 = ... 4*th year.*
130 : 100 : : 100 : 76.9230 = ... 5*th year.*

425.9391 = 425*l.* 18*s.* $9\frac{1}{4}$*d.* = *present worth of the annuity required.*

2. What is the present worth of an annuity or pension of 500*l.* to continue 4 years, at 5 *per cent. per ann.* simple interest? *Ans.* 1782*l.* 5*s.* 7*d.*

To find the Amount of an Annuity at Compound Interest.

RULE.*

1. Make 1 the first term of a geometrical progression, and the amount of 1*l.* for 1 year, at the given rate *per cent.* the ratio.

The other two theorems for the time and rate cannot be given in general terms.

* *Demon.* It is plain, that upon the first year's annuity, there will be due as many years compound interest, as the given number of year's less one, and gradually one year less upon every succeeding year to that preceding the last, which has but one year's interest, and the last bears no interest.

Let r, therefore, = rate, or amount of 1*l.* for 1 year; then the series of amounts of 1*l.* annuity, for several years, from the first to the last, is 1, r, r^2, r^3 &c. to r^{t-1}. And the sum of this, according to the rule in geometrical progression, will be $\frac{r^t-1}{r-1}$,

2. Carry the ſeries to as many terms as the number of years, and find its ſum.

3. Multiply the ſum, thus found, by the given annuity, and the product will be the amount ſought.

EXAMPLES.

1. What is the amount of an annuity of 40 *l.* to continue 5 years, allowing 5 *per cent.* compound intereſt?

$$1 + 1.05 + \overline{1.05}|^{2} + \overline{1.05}|^{3} + \overline{1.05}|^{4} = 5.52563125$$

$$\begin{array}{r} 5.52563125 \\ 40 \\ \hline 221.025250 \\ 20 \\ \hline 0.505000 \\ 12 \\ \hline 6.060000 \end{array}$$

Anſ. 221 *l.* 0 *s.* 6 *d.*

$=$ amount of 1 *l.* annuity for t years. And all annuities are proportional to their amounts, therefore $1 : \frac{r^t - 1}{r - 1} :: n : \frac{r^t - 1}{r - 1} \times n =$ amount of any given annuity n. *Q. E. D.*

Let $r =$ rate, or amount of 1 *l.* for 1 year, and the other letters as before, then

$$\frac{r^t - 1}{r - 1} \times n = a, \quad \textit{and} \quad \frac{ar - a}{r^t - 1} = n;$$

And from theſe equations all the caſes relating to annuities, or penſions in arrears, may be conveniently exhibited in logarithmic terms thus:

I. $\text{Log. } n + \text{Log. } \overline{r^t - 1} - \text{Log. } \overline{r - 1} = \text{Log. } a,$

II. $\text{Log. } a - \text{Log. } \overline{r^t - 1} + \text{Log. } \overline{r - 1} = \text{Log. } n,$

III.

2. If 50 *l.* yearly rent, or annuity, be forborn 7 years, what will it amount to at 4 *per cent. per annum*, compound interest? *Ans.* 395 *l.*

To find the present Value of Annuities at Compound Interest.

RULE.*

1. Divide the annuity by the ratio, or the amount of 1 *l.* for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth of the annuity for 2 years.

3. Find, in like manner, the present worth of each year by itself, and the sum of all these will be the value of the annuity sought.

EXAM-

III. $\dfrac{\text{Log. } \overline{ar - a + n} - \text{Log. } n}{\text{Log. } r.} = t,$

IV. $r^t - \dfrac{ar}{n} + \dfrac{a}{n} - 1 = 0,$

---

* The reason of this rule is evident from the nature of the question, and what was said upon the same subject in the purchasing of annuities by simple interest.

Let p = present worth of the annuity, and the other letters as before, then

$$n + \frac{r^t - 1}{r^{t+1} - r^t} = p, \text{ and } p \times \frac{r^{t+1} - r}{r^t - 1} = n;$$

And from these theorems all the cases, where the purchase of annuities is concerned, may be exhibited in logarithmic terms, as follows:

I. $\text{Log. } n + \text{Log. } \overline{1 - \frac{1}{r}} - \text{Log. } \overline{r - 1} = \text{Log. } p.$

II. $\text{Log. } p + \text{Log. } \overline{r - 1} - \text{Log } \overline{1. - \frac{1}{r^t}} = \text{Log. } n.$

III

EXAMPLES.

1. What is the present worth of an annuity of 40 *l.* to continue 5 years, discounting at 5 *per cent. per annum*, compound interest?

ratio = 1.05)40.00000(38.095 = *present worth for* 1 *year.*

$\overline{ratio}|^2$ = 1.1025)40.00000(36.281 = *do. for* 2 *yrs.*
$\overline{ratio}|^3$ = 1.157525)40.00000(34.556 = *do. for* 3 *yrs.*
$\overline{ratio}|^4$ = 1.215506)40.00000(32.899 = *do. for* 4 *yrs.*
$\overline{ratio}|^5$ = 1.276218)40.00000(31.342 = *do. for* 5 *yrs.*

173.173 = 173 *l.* 3 *s.* 5½ *d.*

= *whole present worth of the annuity required.*

2. What is the present worth of an annuity of 21 *l.* 10 *s.* 9½ *d.* to continue 7 years, at 6 *per cent. per ann.* compound interest? *Ans.* 120 *l.* 5 *s.*

3. What is 70 *l. per annum*, to continue 59 years, worth in present money, at the rate of 5 *per cent. per annum*? *Ans.* 1321.3021 *l.*

To

III. $\dfrac{\text{Log. } n - \text{Log. } \overline{n+p-pr}}{\text{Log. } r} = t.$

IV. $r^t + 1 - \overline{\dfrac{n}{p} + 1} \times rt + \dfrac{n}{p} = 0$

Let t express the number of half years or quarters, n the half year's or quarter's payment, and r the sum of one pound and ½ or ¼ year's interest, then all the preceding rules are applicable to half yearly and quarterly payments the same as to whole years.

The amount of an annuity may also be found for years and parts of a year, thus:

1. Find the amount for the whole years as before.
2. Find the interest of that amount for the given parts of a year.

3. Add

To find the present worth of a freehold estate, or an annuity to continue for ever, at compound interest.

RULE.*

As the rate *per cent.* is to 100 *l.* so is the yearly rent to the value required.

EXAM-

3. Add this interest to the former account and it will give the whole amount required.

The present worth of an annuity for years and parts of a year may be found thus:

1. Find the present worth for the whole years as before.

2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

* The reason of this rule is obvious: for since a year's interest of the price which is given for it is the annuity, there can neither more nor less be made of that price than of the annuity, whether it be employed at simple or compound interest.

The same thing may be shewn thus: The present worth of an annuity to continue for ever, is $\frac{n}{r} + \frac{n}{r^2} + \frac{n}{r^3} + \frac{n}{r^4}$ &c. *ad infinitum*, as has been shewn before; but the sum of this series, by the rules of geometrical progression, is $\frac{n}{r-1}$; therefore $r-1 : 1 :: n : \frac{1}{r-1}$. which is the rule.

The following theorems shew all the varieties of this rule.

I. $\frac{n}{r-1} = p.$

II. $\overline{r-1} \times p. = n.$

III. $\frac{n}{p} + 1 = r$, or $\frac{n}{p} = r - 1$

The price of a freehold estate, or annuity to continue for ever, reckoning simple interest, would be expressed by $\frac{1}{1+r} + \frac{1}{1+2r} + \frac{1}{1+3r} + \frac{1}{1+4r}$ &c. *ad infinitum*; but the sum of this series is infinite,

EXAMPLES.

1. An estate brings in yearly 79 *l.* 4 *s.* what would it sell for, allowing the purchaser $4\frac{1}{2}$ *per cent.* compound interest for his money?

$$
\begin{array}{r}
4.5 \;:\; 100 \;::\; 79.2 \\
100 \\
\hline
4.5)7920.0(1760\,l. \text{ the answer.} \\
45 \\
\hline
342 \\
315 \\
\hline
270 \\
270 \\
\hline
\end{array}
$$

2. What is the price of a perpetual annuity of 40 *l.* discounting at 5 *per cent.* compound interest? *Ans.* 800 *l.*

3. What is a freehold estate of 75 *l.* a year worth, allowing the buyer 6 *per cent.* compound interest for his money? *Ans.* 1250 *l.*

To find the present worth of an annuity, or freehold estate, in reversion, at compound interest.

RULE.*

1. Find the present worth of the annuity as though it were to be entered on immediately.

2. Find

infinite, or greater than any assignable number, which sufficiently shews the absurdity of using simple interest in these cases.

* This rule is sufficiently evident without a demonstration.

Those who wish to be acquainted with the manner of computing the values of annuities upon lives, may consult the writings of Mr. *Demoivre*, Mr. *Simpson* and Dr. *Price*, all of whom have handled this subject in a very skilful and masterly manner.

Dr.

2. Find the present worth of the last present worth, discounting for the time between the purchase and commencement of the annuity, and it will be the answer required.

EXAMPLES.

1. The reversion of a freehold estate of 79 *l.* 4 *s. per annum*, to commence 7 years hence, is to be sold, what is it worth in ready money, allowing the purchaser 4 ½ *per cent.* for his money?

```
4.5 : 100 :: 79.2
               100
            -------
      4.5)7920.0(1760 = present worth
          45            if entered on immedi-
          ----          ately.
           342
           315
           ----
            270
            270
            ----
              0
```

and $\overline{1.045}|^7 = 1.360862)1760.000(1293.297 = 1293$ *l.* 5 *s.* 11¼ *d.* = *present worth of* 1760 *l. for* 7 *years, or the whole present worth required.*

2. Suppose an estate is worth 20 *l. per annum*, and a fine of 100 *l.* for a lease of 21 years. Now, if the fine be dropped, how much ought the rent to be increased, allowing 5 *per cent.* compound interest? *Ans.* 7 *l.* 16 *s.*

3. Which is most advantageous a term of 15 years in an estate of 100 *l. per annum*, or the reversion of such an

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Dr. *Price*'s treatise upon annuities and reversionary payments is an excellent performance, and will be found a very valuable acquisition to those whose inclinations lead them to studies of this nature.

an estate for ever, after the expiration of the said 15 years, computing at the rate of 5 *per cent. per ann.* compound interest? *Ans. The first term of* 15 *years is better than the reversion for ever afterwards by* 75*l.* 18*s.* 7½*d.*

4. Suppose I would add 5 years to a running lease of 15 years to come, the improved rent being 186 *l.* 7 *s.* 6 *d. per ann.*; what ought I to pay down for this favour, discounting at 4 *per cent. per ann.* compound interest?

Ans. 460 *l.* 14 *s.* 1¾*d.*

ARITHMETICAL PROGRESSION.

Any rank of numbers increasing by a common excess, or decreasing by a common difference, are said to be in *arithmetical progression*; such are the numbers 1, 2, 3, 4, 5 &c. and 7, 5, 3, 1, .8, .6 &c.

The numbers which form the series are called the *terms* of the progression.

Any three of the five following terms being given, the other two may be readily found.

1. The first term, } commonly called the *extremes*.
2. The last term, }
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM I.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

RULE.*

Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAM-

* Suppose another series of the same kind with the given one be placed under it in an inverse order; then will the sum of every two

EXAMPLES.

1. The firſt term of an arithmetical progreſſion is 2, the laſt term 53, and the number of terms 18, required the ſum of the ſeries.

$$\begin{array}{r} 53 \\ 2 \\ \hline 55 \\ 18 \\ \hline 440 \\ 55 \\ \hline 2)990 \\ \hline 495 \end{array}$$

Or, $\dfrac{\overline{53+2}\times 18}{2} = 495$ *the anſwer.*

2. The firſt term is 1, the laſt term 21, and the number of terms 11, required the ſum of the ſeries.

Anſ. 121

3. How many ſtrokes do the clocks of Venice, which go to 24 o'clock, ſtrike in the compaſs of a day?

Anſ. 300

4. If 100 ſtones are placed in a right line, exactly a yard aſunder, and the firſt a yard from a baſket; what length

---

two correſponding terms be the ſame as that of the firſt and laſt; conſequently any one of thoſe ſums multiplied by the number of terms muſt give the whole ſum of the two ſeries, and half that ſum will, evidently, be the ſum of the given ſeries: thus,

Let 1. 2. 3. 4. 5. 6. 7. be the given ſeries.
and 7. 6. 5. 4. 3. 2. 1. the ſame inverted,
then $8+8+8+8+8+8+8 = 8\times 7 = 56$ and $1+3+4+5+6+7 = \frac{56}{2} = 28$.

Q. E. I.

length of ground will that man go who gathers them up singly, returning with them one by one to the basket? *Ans.* 5 *miles and* 1300 *yards.*

PROBLEM 2.

The first term, the last term, and the number of terms being given, to find the common difference.

RULE.*

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

EXAMPLES.

1. The extremes are 2 and 53, and the number of terms is 18, required the common difference.

$$\begin{array}{r} 53 \\ 2 \\ \hline 17)51(3 \\ 51 \\ \hline \end{array} \qquad \begin{array}{r} 18 \\ 1 \\ \hline 17 \end{array}$$

Or

$$\frac{53-2}{18-1} = \frac{51}{17} = 3 \textit{ the answer.}$$

2. If the extremes be 3 and 19, and the number of terms 9; it is required to find the common difference, and the sum of the whole series.

Ans. The diff. is 2, *and the sum is* 99

3. A man

---

* The difference of the first and last terms evidently shows the increase of the first term, by all the subsequent additions, till it becomes equal to the last; and as the number of those additions were evidently one less than the number of terms, and the increase by every addition equal, it is plain that the total increase divided by the number of additions must give the difference at every one separately: whence the rule is manifest.

3. A man is to travel from London to a certain place in 12 days, and to go but 3 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 58 miles; required the daily increase, and the distance of the place from London.

Ans. Daily increase 5, *distance* 366 *miles.*

PROBLEM 3.

Given the first term, the last term, and the common difference to find the number of terms.

RULE.*

Divide the difference of the extremes by the common difference, and the quotient increased by 1 is the number of terms required.

EXAMPLES.

1. The extremes are 2 and 53, and the common difference 3, what is the number of terms?

$$\begin{array}{r} 53 \\ 2 \\ \hline 3)51 \\ \hline 17 \\ 1 \\ \hline 18 \end{array}$$

Or, $\frac{53-2}{3} + 1 = 18$ *the answer.*

2. If

* By the last problem the difference of the extremes divided by the number of terms less one, gives the common difference; consequently the same divided by the common difference must give the number

2. If the extremes be 3 and 19, and the common difference 2; what is the number of terms? *Anſ.* 9

3. A man going a journey, travelled the firſt day 5 miles, the laſt day 35 miles, and increaſed his journey every day by 3 miles; how many days did he travel? *Anſ.* 11 *days.*

---

number of terms leſs one; hence this quotient augmented by one muſt be the anſwer to the queſtion.

In any arithmetical progreſſion, the ſum of any two of its terms is equal to the ſum of any other two terms taken at an equal diſtance, on contrary ſides of the former; or the double of any one term, is equal to the ſum of any two terms taken at an equal diſtance from it on each ſide.

The ſun of any number of terms (n) of the arithmetical ſeries of odd numbers 1, 3, 5, 7, 9, &c. is equal to the ſquare (n^2) of that number.

That is, if 1, 3, 5, 7, 9, &c. be the numbers.

Then will 1, 2^2, 3^2, 4^2, 5^2, &c. be the ſums of 1, 2, 3 and of thoſe terms.

For, $0 + 1$ or the ſum of 1 term $= 1^2$ or 1
$1 + 3$ or the ſum of 2 terms $= 2^2$ or 4
$4 + 5$ or the ſum of 3 terms $= 3^2$ or 9
$9 + 7$ or the ſum of 4 terms $= 4^2$ or 16 &c.

Whence it is plain, that, let n be any number whatſoever, the ſum of n terms will be n^2.

The following table contains a ſummary of the whole doctrine of arithmetical progreſſion.

CASES

Cases *of* Arithmetical Progression.

Case	*Giv.*	*Req.*	*Solution.*
1.	*adn*	*l*	$\overline{n-1}\times d+a$
		s	$n\times\overline{a+\overline{n-1}\times\frac{d}{2}}$
2.	*adl*	*n*	$\frac{l-a}{d}+1.$
		s	$\frac{\overline{l+a}\times\overline{l-a+d}}{2d}$
3.	*ads*	*n*	$\frac{\sqrt{\overline{2a-d}\rvert^{2}+8ds}-\overline{2a-d}}{2d}$
		l	$\frac{\sqrt{\overline{2a-d}\rvert^{2}+8ds}-d}{2}$
4.	*als*	*d*	$\frac{\overline{l+a}\times\overline{l-a}}{2s-\overline{l+a}}$
		n	$\frac{2s}{a+l}$
5.	*ans*	*d*	$\frac{2\times\overline{s-an}}{\overline{n-1}\times n}$
		l	$\frac{2s}{n}-a.$

Case	Giv.	Req.	Solution.
6.	aln	d	$\frac{l-a}{n-1}$
		s	$\frac{\overline{a+l}\times n}{2}$
7.	dnl	a	$l-\overline{n-1}\times d$
		s	$n\times\overline{l-\overline{n-1}\times\frac{d}{2}}$
8.	snd	a	$\frac{s}{n}-\frac{d\times\overline{n-1}}{2}$
		l	$\frac{s}{n}-\frac{d\times\overline{n-1}}{2}$
9.	dls	a	$\frac{d\pm\sqrt{\overline{2l+d}\vert^{2}-8ds}}{2}$
		n	$\frac{2l+d\mp\sqrt{\overline{2l+d}\vert^{2}-8ds}}{2d}$
10.	lns	a	$\frac{2s}{n}-l.$
		d	$\frac{2\times\overline{nl-s}}{\overline{n-1}\times n}$

Here:
- a = *least term*
- n = *number of terms*
- s = *sum of all the terms*
- d = *common difference*
- l = *greatest term.*

GEOMETRICAL PROGRESSION.*

Any ſeries of numbers, the terms of which gradually increaſe or decreaſe by a conſtant multiplication or diviſion, is ſaid to be in *geometrical progreſſion.* Thus, 4, 8, 16, 32, 64 &c. and 81, 27, 9, 3, 1 &c. are ſeries in geometrical progreſſion, the one increaſing by a conſtant multiplication by 2, and the other decreaſing by a conſtant diviſion by 3.

The number by which the ſeries is conſtantly increaſed or diminiſhed is called the *ratio.*

PROBLEM 1.

Given the firſt term, the laſt term, and the ratio, to find the ſum of the ſeries.

---

* Numbers are compared together to diſcover the relations they have to each other.

There muſt always be two numbers to form a compariſon: the number which is compared, being written firſt, is called the antecedent, and that to which it is compared the conſequent. Thus, if 3 : 6 : : 12 : 24, 3 and 12 are called the antecedents, and 6 and 24 the conſequents. And when the terms of two ratios, making a proportion, ſucceed one another in the manner of a geometrical progreſſion, they are ſaid to be in *continued* geometrical proportion; but when the proportion is broken, or the ratios are taken between ſuch pairs of numbers as do not ſtand together in a geometrical progreſſion, the proportion is ſaid to be *diſcontinued*: Thus, 2 : 4 : : 8 : 16 is in continued proportion, and 2 : 3 : : 10 : 15 in diſcontinued proportion.

Three or four quantities are ſaid to be in *harmonical proportion*, when, in the former caſe, the difference of the firſt and ſecond is to the difference of the ſecond and third as the firſt is to the third; and, in the latter, when the difference of the firſt and ſecond is to the difference of the third and fourth as the firſt is to the fourth. Thus 2, 3 and 6, and 3, 4, 6, 9 are harmonical proportionals.

Four numbers are ſaid to be *reciprocally* or *inverſely proportional*, when the fourth is leſs than the ſecond by as many times as the third is greater than the firſt, or when the firſt is to the third as the fourth to the ſecond, and *vice verſa.* Thus, 2, 9, 6 and 3 are reciprocal proportionals.

RULE.*

Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by the ratio less one will give the sum of the series.

EXAM-

If $a : b :: c : d$ directly.

$$\text{Then}\begin{cases} a : c :: b : d \text{ by alternation.} \\ b : a :: d : c \text{ by inversion.} \\ a+b : b :: c+d : d \text{ by composition.} \\ a-b : b :: c-d : d \text{ by division.} \\ a : a+b :: c : c+d \text{ by conversion.} \\ a+b : a-b :: c+d : c-d \text{ mixedly.} \end{cases}$$

* In order to demonstrate the truth of the rule I shall premise the following Lemmas.

LEMMA 1.

In any geometrical progression of three terms, the square of the mean term is equal to the product of the extremes. Thus, in 2, 6, 18, it will be $2 \times 18 = 6^2 = 36$, and the same of any series of three terms.

Demon. It is plain, that in any geometrical series of three terms, the last term will always be equal to the square of the ratio multiplied into the first term; and the second term equal to the first multiplied by the ratio; consequently as the component factors of the product of the extremes are constantly the same as those of the square of the mean, the results of each must be equal. Thus, in the example above, the last term is equal to $3 \times 3 \times 2$, which multiplied by the first is $3 \times 3 \times 2 \times 2 = 36$; and the second term is 3×2, which squared is $3 \times 3 \times 2 \times 2 = 36$. *Q. E. D.*

Coroll. The middle term is called a geometrical mean between the two extremes, and is always equal to the square root of their product.

LEMMA 2.

In any geometrical series of four terms, the product of the two means is equal to that of the two extremes.—Thus, if $3 : 6 :: 12 : 24$, $3 \times 24 = 6 \times 12$.

Demon. It is plain, from the nature of multiplication, that if one factor be increased as many times as the other is diminished, their product will still be the same. Hence, in the above series, as 6 exceeds 3 as many times as 24 exceeds 12, it is manifest, from what was

EXAMPLES.

1. The first term of a series in geometrical progression is 1, the last term is 2187, and the ratio 3: what is the sum of the series?

$$\begin{array}{r} 2187 \\ 3 \\ \hline 6561 \\ 1 \\ \hline 3-1=2)6560 \\ \hline 3280 \end{array}$$

$$\text{Or, } \frac{3 \times 2187 - 1}{3 - 1} = 3280 \textit{ the answer.}$$

---

was said in the demonstration of the preceding Lemma, that the product of the extremes will always be equal to that of the means. *Q. E. D.*

Coroll. In any geometrical series consisting of an even number of terms, the product of the means will be equal to the product of the extremes, or any other pair equally distant from them.

And if the series contain an odd number of terms, the square of the mean will be equal to the product of the adjoining extremes, or any two equally distant from them.

Demon. of the rule. Take any series whatever, as 1. 3. 9. 27. 81. 243 &c. multiply this by the ratio, and it will produce the series 3. 27. 81. 243. 729 &c. Now, let the sum of the proposed series be what it will, it is plain, that the sum of the second series will be as many times the former sum as is expressed by the ratio; subtract the first series from the second, and it will give 729—1: which is evidently as many times the sum of the first series as is expressed by the ratio less one; consequently $\frac{729-1}{3-1}$ = sum of the proposed series, and is the rule; or 729 is the last term multiplied by the ratio, 1 is the first term, and 3—1 is the ratio less one; and the same will hold let the series be what it will. *Q. E. D.*

2. The extremes of a geometrical progression are 1 and 65536, and the ratio 4: what is the sum of the series? *Ans.* 87381

3. The extremes of a geometrical series are 1024 and 59049, and the ratio is $1\frac{1}{2}$: what is the sum of the series? *Ans.* 175099

PROBLEM 2.

Given the first term and the ratio, to find any other term assigned.

RULE.*

1. Write down a few of the leading terms of the series, and place their indices over them, beginning with a cypher.

2. Add together the most convenient indices to make an index less by one than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power whose index is one less than the number of terms multiplied, and make the result a divisor.

5. Divide the dividend by the divisor, and the quotient will be the term sought.

---

* *Demon.* In example 1st, where the first term is equal to the ratio, the reason of the rule is evident; for as every term is some power of the ratio, and the indices point out the number of factors, it is plain from the nature of multiplication, that the product of any two terms, will be another term corresponding with the index, which is the sum of the indices standing over those respective terms.

And in the second example, where the series doth not begin with the ratio, it appears that every term, after the two first, contains some power of the ratio multiplied into the first term, and therefore the rule, in this case, is equally evident.

The table in page 174 contains all the possible cases of geometrical progression.

Note.

Note. When the firſt term of the ſeries is equal to the ratio the indices muſt begin with an unit; and, in this caſe, the product of the different terms, found as before, will give the term required.

EXAMPLES.

1. The firſt term of a geometrical ſeries is 2, the number of terms 13, and the ratio 2; required the laſt term.

1.	2.	3.	4.	5	*indices*
2.	4.	8.	16.	32	*leading terms.*

Then $4+4+3+2=$ *index to* 13*th. term.*
And $16\times16\times8\times4=8192$ *the anſwer.*

In this example the indices muſt begin with 1, *and ſuch of them be choſen as will make up the entire index to the term required.*

2. Required the 12th. term of a geometrical ſeries, whoſe firſt term is 3 and ratio 2.

0.	1.	2.	3.	4.	5.	6	*indices*
3.	6.	12.	24.	48.	96.	192	*leading terms.*

Then $6+5=$ *index to* 12*th term.*
and $192\times96=18432=$ *dividend.*

The number of terms multiplied is 2, *and* $2-1=1$, *is the power to which the term* 3 *is to be raiſed; but the* 1*ſt power of* 3 is 3, *and therefore* $18432\div3=6144$ *the* 12*th term required.*

3. The firſt term of a geometric ſeries is 1, the ratio 2, and the number of terms 23; required the laſt term. *Anſ.* 4194304

4. A perſon being aſked to diſpoſe of a fine horſe, ſaid he would ſell him on condition of having one farthing for the firſt nail in his ſhoes, 2 farthings for the ſecond, one penny for the third, and ſo on, doubling the price of every nail to 32, the number of nails in his four ſhoes: what would the horſe be ſold for at that rate? *Anſ.* 4473924*l.* 5*s.* $3\frac{3}{4}$*d.*

Cases of Geometrical Progression.

Case	Giv.	Req.	Solution.
1.	arn	l	ar^{n-1}
		s	$\frac{r^n-1}{r-1}\times a$
2.	arl	s	$l+\frac{l-a}{r-1}$
		n	$\frac{L,l-L,a}{L,r}+1$
3.	ars	l	$\frac{\overline{r-1}\times s+a}{r}$
		n	$\frac{L,\overline{\overline{r-1}\times s+a}-L,a}{L,r}$
4.	als	r	$\frac{s-a}{s-l}$
		n	$\frac{L,l-L,a}{L,\overline{s-a}-L,\overline{s-l}}+1$
5.	ans	r	$r^n-\frac{rs}{a}=\frac{a-s}{a}$
		l	$l\times\overline{s-l}\vert^{n-1}=a\times\overline{s-a}\vert^{n-1}$

Case

Case	Giv.	Req.	Solution
6.	anl	r	$\overline{\frac{l}{a}}\Big\|^{\frac{1}{n-1}}$
		s	$l + \dfrac{l-a}{\overline{\frac{l}{a}}\Big\|^{\frac{1}{n-1}} - 1}$
7.	rnl	a	$\dfrac{l}{r^{n-1}}$
		s	$l + \dfrac{l - \dfrac{l}{r^{n-1}}}{r-1}$
8.	rns	a	$\dfrac{r-1}{r^n-1} \times s$
		l	$\dfrac{r^n - r^{n-1}}{r^n-1} \times s$
9.	rls	a	$s - r \times \overline{s-l}$
		n	$\dfrac{L,l - L,\overline{s - r\times\overline{s-l}}}{L,r} + 1$
10.	nls	a	$a\times\overline{s-a}\|^{n-1} = l\times\overline{s-l}\|^{n-1}$
		r	$r^n + \dfrac{s}{l-s}\, r^{n-1} = \dfrac{l}{l-s}$

Here $\left\{\begin{array}{l} a = \textit{least term} \\ l = \textit{greatest term} \\ s = \textit{sum of all the terms} \\ n = \textit{number of terms} \\ r = \textit{ratio} \\ L = \textit{Logarithm} \end{array}\right.$

INVOLUTION:

Or the RAISING *of* POWERS.

A *power* is the product arising from multiplying any given number into itself continually a certain number of times, thus,

$2 \times 2 = 4$ is the 2d. power, or square of 2.
$2 \times 2 \times 2 = 8$ is the 3d. power, or the cube of 2.
$2 \times 2 \times 2 \times 2 = 16$ is the 4th. power of 2, &c.

The number denoting the power is called the *index*, or the *exponent* of that power.

If two or more powers are multiplied together, their product is that power whose index is the sum of the exponents of the factors: thus,

$2 \times 2 = 4$ the square of 2; $4 \times 4 = 16 =$ 4th. power of 2; and $16 \times 16 = 256 =$ 8th. power of 2. &c.

EXAMPLES.

1. What is the 6th power of 7?

$$
\begin{array}{r l}
7 & \\
7 & \\
\hline
49 & = \textit{2d. power.} \\
7 & \\
\hline
343 & = \textit{3d. power.} \\
7 & \\
\hline
2401 & = \textit{4th power.} \\
7 & \\
\hline
16807 & = \textit{5th. power.} \\
7 & \\
\hline
117649 & = \textit{6th power, or answer.}
\end{array}
$$

2. What

2. What is the 3d. power of 35? *Ans.* 42875
3. What is the 4th. power of $\frac{3}{4}$? *Ans.* $\frac{81}{256}$
4. What is the 5th. power of .029?

Ans. 00000002051149

EVOLUTION:

Or the EXTRACTING *of* ROOTS.

The *root* is a number, whose continual multiplication into itself produces the power, and is denominated the square, cube, 4th. 5th. root &c. according as it is, when raised to the 2d. 3d. 4th. 5th &c. power, equal to that power. Thus 2 is the square root of 4, because $2 \times 2 = 4$; and 4 is the cube root of 64, because $4 \times 4 \times 4 = 64$; and so on.

Although there is no number of which we cannot find any power exactly, yet there may be many numbers of which a precise root can never be determined. But, by the help of decimals, we can approximate towards the root, to any assigned degree of exactness.

The roots which approximate are called *surd roots*, and those which are perfectly accurate are called *rational roots*.

Roots are sometimes denoted by writing the character $\surd$ before the power, with the index of the root against it: thus, the third root of 70 is expressed $\sqrt[3]{70}$, and the second root of it is $\sqrt{70}$, the index 2 being always omitted when the square root is designed.

If the power be expressed by several numbers, with the sign + or — between them, a line is drawn from the top of the sign over all the parts of it; thus, the third root of 28 — 13 is $\sqrt[3]{28-13}$.

Sometimes roots are designed like powers, with fractional indices; thus, the square root of 5 is $5^{\frac{1}{2}}$, the third root of 19 is $19^{\frac{1}{3}}$, and the fourth root of 40—12 is $\overline{40-12}^{\frac{1}{4}}$ &c.

To EXTRACT THE SQUARE ROOT.

RULE.*

1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on.

2. Find

---

* In order to shew the reason of the rule, it will be proper to premise the following

Lemma. The product of any two numbers can have at most but as many places of figures as are in both the factors, and at least but one less.

Demon. Take two numbers, consisting of any number of places, but let them be the least possible of those places, *viz.* unity with cyphers, as 1000 and 100; then their product will be 1 with as many cyphers annexed as are in both the numbers, *viz.* 100000; but 100000 has one place less than 1000 and 100 together have; and since 1000 and 100 were taken the least possible, the product of any other two numbers, of the same number of places, will be greater than 100000; consequently the product of any two numbers can have, at least, but one place less than both the factors.

Again, take two numbers, of any number of places, that shall be the greatest possible of those places, as 999 and 99. Now 999×99 is less than 999×100; but 999×100 ($=99900$) contains only as many places of figures as are in 999 and 99; therefore 999×99, or the product of any other two numbers consisting of the same number of places, cannot have more places of figures than are in both its factors.

Coroll. 1. A square number cannot have more places of figures than double the places of the root, and, at least, but one less.

Coroll. 2. A cube number cannot have more places of figures than triple the places of the root, and, at least, but two less.

The truth of the rule may be shewn algebraically, thus:

Let N = number whose square root is to be found.

Now, it appears from the lemma, that there will be always as many places of figures in the root as there are points or periods in the given number, and therefore the figures of those places may be represented by letters.

Suppose N to consist of two periods, and let the figures in the root be represented by a and b.

Then

2. Find a square number either equal to, or the next less than the first period, and put the root of it to the right hand of the given number, after the manner of a quotient figure in division, and it will be the first figure of the root required.

3. Subtract the assumed square from the first period, and to the remainder bring down the next period for a dividend.

4. Place the double of the root, already found, on the left hand of the dividend, for a divisor.

5. Consider what figure must be annexed to the divisor, so that if the result be multiplied by it, the pro-

---

Then $\overline{a+b}^2 = a^2 + 2ab + b^2 = N =$ given number; and to find the root of N is the same as finding the root of $a^2 + 2ab + b^2$, the method of doing which is as follows:

1st. divisor a) $a^2 + 2ab + b^2$ ($a + b =$ root.

a^2

2d. divisor $2a + b$) $2ab + b^2$

$2ab + b^2$

Again, suppose N to consist of 3 periods, and let the figures of the root be represented by a, b and c.

Then $\overline{a+b+c}^2 = a^2 + 2ab + b^2 + 2ac + 2bc + c^2$, and the manner of finding a, b and c will be as before: thus,

1st. divisor a) $a^2 + 2ab + b^2 + 2ac + 2bc + c^2$ ($a + b + c =$ root.

a^2

2d. divisor $2a + b$) $2ab + b^2$

$2ab + b^2$

3d. divisor $2a + 2b + c$) $2ac + 2bc + c^2$

$2ac + 2bc + c^2$

Now, the operation, in each of these cases, exactly agrees with the rule, and the same will be found to be true when N consists of any number of periods whatever.

duct may be equal to, or the next less than the dividend, and it will be the second figure of the root.

6. Subtract the product from the dividend, and to the remainder bring down the next period, for a new dividend.

7. Find a divisor as before, by doubling the figures already in the root; and from these find the next figure of the root, as in the last article; and so on through all the periods to the last.

Note, if there are decimals in the given number, it must be pointed both ways from unity, and the root be made to consist of as many whole numbers and decimals as there are periods belonging to each; and when the figures belonging to the given number are exhausted, the operation may be continued at pleasure by adding cyphers.

EXAMPLES.

1. Required the square roots of 5499025, and 184.2.

```
      5499025(2345 the root.
      4
    43)149
       129
   464)2090
       1856
  4685)23425
       23425

      184.2000(13.57 the root.
      1
    23)84
       69
   265)1520
       1325
  2707)19500
       18949
         551 remainder.
```

2. What

2. What is the ſquare root of 106929? *Anſ.* 327
3. What is the ſquare root of 152399025?
Anſ. 12345
4. What is the ſquare root of 119550669121?
Anſ. 345761
5. What is the ſquare root of 368863?
Anſ. 607.34092 &c.
6. What is the ſquare root of 3.1721812?
Anſ. 1.78106 &c.
7. What is the ſquare root of .00032754?
Anſ. .01809
8. What is the ſquare root of $\frac{5}{12}$? *Anſ.* .645497
9. What is the ſquare root of $6\frac{2}{5}$? *Anſ.* 2.5298 &c.
10. What is the ſquare root of 10?
Anſ. 3.162277 &c.

THE EXTRACTION OF THE CUBE ROOT.

RULE 1.*

1. Separate the given number into periods of three figures each, by putting a point over every third figure from the place of units.

2. Find the greateſt cube in the firſt period, and put its root in the quotient.

3. Subtract the cube thus found from the ſaid period, and to the remainder prefix the next period, and call this the *reſolvend.*

4. Under this reſolvend write the triple ſquare of the root, ſo that units in the latter may ſtand under the place of hundreds in the former; and under the ſaid triple

* The reaſon of pointing the given number, as directed in the rule, is obvious from Coroll. 2. to the lemma made uſe of in demonſtrating the ſquare root; and the reſt of the operation will be beſt underſtood from the following analytical proceſs:

Suppoſe

triple square write the triple root, removed one place to the right hand, and call the sum of these the *divisor*.

5. Seek how often the divisor may be had in the resolvend, exclusive of the place of units, and write the result in the quotient.

6. Under the divisor write the product of the triple square of the root by the last quotient figure, setting down the units place of this line, under the place of tens in the divisor; under this line write the product of the triple root by the square of the last quotient figure, so as to be removed one place beyond the right-hand figure of the former; and under this line, removed one place forward to the right-hand, write down the cube

---

* Suppose N, the given number, to consist of two periods, and let the figures in the root be denoted by a and b.

Then $\overline{a+b}|^3 = a^3 + 3a^2b + 3ab^2 + b^3 = \text{N} =$ given number, and to find the cube root of N is the same as to find the cube root of $a^3 + 3a^2b + 3ab^2 + b^3$; the method of doing which is as follows:

$$
\begin{array}{llll}
a^3 + 3a^2b + 3ab^2 + b^3 \;(a + b = \textit{root}. \\
a^3 \\
\hline
\quad 3a^2b + 3ab^2 + b^3 \; \textit{resolvend}. \\
\hline
\quad 3a^2 \\
\qquad + 3a \\
\hline
\quad 3a^2 + 3a \; \textit{divisor}. \\
\hline
\quad 3a^2b \\
\qquad + 3ab^2 \\
\qquad\qquad + b^3 \\
\hline
\quad 3a^2b + 3ab^2 + b^3 \; \textit{subtrahend}. \\
\hline
\qquad * \quad *
\end{array}
$$

And in the same manner may the root of a quantity consisting of any number of periods whatever be found.

of the laſt quotient figure, and call their ſum the *ſubtrahend*.

7. Subtract the ſubtrahend from the reſolvend, and to the remainder bring down the next period for a new reſolvend, with which proceed as before, and ſo on till the whole is finiſhed.

Note. The ſame rule muſt be obſerved for continuing the operation, and pointing for decimals, as in the ſquare root.

EXAMPLES.

1. Required the cube root of 48228544.

48228544(364
27

21228 *reſolvend.*

27 *triple ſquare of* 3.
9 *triple of* 3.

279 *diviſor.*

162 *triple ſquare of* 3, *multiplied by* 6.
324 *triple of* 3, *multiplied by the ſquare of* 6.
216 *cube of* 6.

19656 *ſubtrahend.*

1572544 *ſecond reſolvend.*

3888 *triple ſquare of* 36.
108 *triple of* 36.

38988 *ſecond diviſor.*

15552 *triple ſquare of* 36, *multiplied by* 4.
1728 *triple of* 36, *multiplied by the ſquare of* 4.
64 *cube of four.*

1572544 *ſecond ſubtrahend.*

* *

2. What is the cube root of 389017? *Ans.* 73
3. What is the cube root of 1092727? *Ans.* 103
4. What is the cube root of 27054036008?
Ans. 3002
5. Required the cube root of 122615327232.
Ans. 4968
6. What is the cube root of 146708.483? *Ans.* 52.74
7. What is the cube root of 171.46776406?
Ans. 5.555 &c.
8. What is the cube root of .0001357?
Ans. 05138 &c.
9. Extract the cube root of $13\frac{2}{3}$. *Ans.* 2.3908
10. What is the cube root of $\frac{15120}{51130}$ *Ans.* $\frac{2}{3}$
11. What is the cube root of $\frac{2}{3}$? *Ans.* .873 &c.

RULE 2.*

1. Find, by trial, a cube near to the given number, and call it the ſuppoſed cube.

2. Then, twice the ſuppoſed cube added to the given number, is to twice the given number added to the ſuppoſed cube, as the root of the ſuppoſed cube is to the root required.

3. By taking the cube of the root thus found for the ſuppoſed cube, and repeating the operation, the root will be had to a ſtill greater degree of exactneſs.

EXAM-

* The methods uſually given for extracting the cube root are ſo exceedingly tedious and difficult to be remembered, that arithmeticians have long wiſhed for a ſhort eaſy rule that would be more ready and convenient in practice. Sir *Iſaac Newton*, Dr. *Halley*, Mr. *Simpſon*, Mr. *Emerſon*, and ſeveral other mathematicians of the greateſt eminence, have invented approximating rules for this purpoſe; but no one, that I have yet ſeen, is ſo ſimple in its form, or ſeems ſo well adapted for general uſe as that given above.

That it converges extremely faſt may be eaſily ſhewn, as follows:

Let

EXAMPLES.

1. It is required to find the cube root of 98003449.

Let 125000000 = *supposed cube, whose root is* 500;

$$
\begin{array}{llllll}
\textit{Then} & 125000000 & & 98003449 & & \\
 & \underline{\qquad\qquad 2} & & \underline{\qquad\qquad 2} & & \\
 & 250000000 & & 196006898 & & \\
 & \underline{\ \ 98003449} & & \underline{125000000} & & \\
 & 348003449 & : & 321006898 & :: & 500 \\
 & & & \underline{\qquad\qquad 500} & & \\
\end{array}
$$

$$
\begin{array}{l}
348003449)160503449000(461 = \textit{corrected [root} \\
\qquad\qquad\qquad\ \underline{1392013796} \\
\qquad\qquad\qquad\ \ 2130206940 \\
\qquad\qquad\qquad\ \ \underline{2088020694} \\
\qquad\qquad\qquad\qquad 412862460 \\
\qquad\qquad\qquad\qquad \underline{348003449} \\
\qquad\qquad\qquad\qquad\ \ 73859011 \\
\end{array}
$$

Again,

Let N = given number, a^3 = supposed cube, and x = correction.

Then $\overline{2a^3 + N} : \overline{2N + a^3} :: a : \overline{a + x}$ by the rule, and consequently $\overline{2a^3 + N} \times \overline{a + x} = \overline{2N + a^3} \times a$,

or $\overline{2a^3 + \overline{a + x}^3} \times \overline{a + x} = \overline{2N + a^3} \times a$.

Or $2a^4 + 2a^3x + a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4 = 2aN + a^4$, and by transposing the terms, and dividing by $2a$

$N = a^3 + 3a^2x + 3ax^2 + x^3 + x^3 + \frac{x^4}{2a}$, which by neglecting the terms $x^3 + \frac{x^4}{2a}$, as being very small, becomes N

$= a^3$

Again, let 97972181 = *supposed cube, whose root is* 461.

```
Then 97972181            98003449
            2                   2
    ---------            ---------
    195944362            196006898
     98003449             97972181
    ---------            ---------
    293947811   :        293979079    :  :  461
                               461
                         ---------
                         293979079
                       1763874474
                      1175916316
                     ------------
     293947811)135524355419(461.04903778 nearly
                1175791244
               -----------
                1794523101
                1763686866
               -----------
                  308362359
                  293947811
                 ----------
                   1441454800
                   1175791244
                   ----------
                    2656635560
                    2645530299
                    ----------
                      1110526100
                       881843433
                      ----------
                       2286826670
                       2057634677
                       ----------
                        2291919930
                        2057634670
                        ----------
                         234285260
```

2. What

$= a^3 + 3a^2x + 3ax^2 + x^3 =$ to the known cube of $a + x$.

Q. E. I.

This rule I received from Mr. *Reuben Robbins*, who informs me that he had it from the late Mr. *James Dodson* at the time he was mathematical master of Christ's Hospital.

2. What is the cube root of 157464? *Ans.* 54

3. What is the cube root of 164566592? *Ans.* 548

4. What is the cube root of 673373097125?
Ans. 8765

5. What is the cube root of 7121.1021698?
Ans. 19.238 &c.

6. What is the cube root of $\frac{4}{9}$? *Ans.* .763 &c.

7. What is the cube root of .0069761218?
Ans. .19107 &c.

8. What is the cube root of 117? *Ans.* 4.89097

To EXTRACT THE ROOTS OF POWERS IN GENERAL.

R U L E.*

1. Prepare the given number for extraction, by pointing off from the units place as the root required directs.

2. Find the first figure of the root by trial, and subtract its power from the given number.

3. To the remainder bring down the first figure in the next period, and call it the *dividend*.

2. In-

* This rule will be sufficiently obvious from the work in the following example:

Extract the cube root of $a^6+6a^5-40a^3+96a-64$.

$$
\begin{array}{l}
\qquad\qquad\qquad a^6+6a^5-40a^3+96a-64\,(a^2+2a-4 \\
\qquad\qquad\qquad a^6 \\
\qquad\qquad\qquad \overline{} \\
\qquad\qquad\qquad 3a^4)\,6a^5\,(+2a \\
\qquad\qquad\qquad \overline{a^6+6a^5+12a^4+8a^3} = \overline{a^2+2a}^{\,3} \\
\overline{a^2+2a}^{\,2}\times 3 = 3a^4+12a^3+12a^2)\,-12a^4-48a^3+96a-64\,(-4 \\
\qquad\qquad\qquad \overline{a^6+6a^5-40a^3+96a^2-64} = \overline{a^2+2a-4}^{\,3} \\
\qquad\qquad\qquad\qquad * \qquad\qquad *
\end{array}
$$

The

4. Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the given number as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, and so on till the whole is finished.

EXAMPLES.

1. What is the cube root of 53157376?

53157376(376?
$27 = 3^3$

$3^2 \times 3 = 27$)261 *dividend*

$50653 = 37^3$

$3^2 \times 3 = 4107$)25043 *second dividend*
53157376

0

2. What is the biquadrate root of 19987173376?

Ans. 376

3. Extract

The extracting of the roots of very high powers by this rule will be found a tedious operation, and will be made use of only by those who are not acquainted with other methods.

When the index of the power whose root is to be subtracted is a composite number, the following rule will be serviceable:

Take any two or more indices, whose product is the given index, and extract out of the given number a root answering to one of these indices;

3. Extract the ſurſolid, or fifth root, of 307682821106715625. *Anſ.* 3145

4. Extract the ſquare cubed, or ſixth root, of 435728381009267809889;64416 *Anſ.* 27534

5. Find the ſeventh root of 3448771746730751318249215379467 3. *Anſ.* 32017

6. Find the eighth root of 11210162813204762362464979424604 81. *Anſ.* 13527

7. Find the ninth root of 976379602989073960279630298890. *Anſ.* 2148.7201

POSITION.

Poſition is a method of performing ſuch queſtions as cannot be reſolved by the common direct rules~~, and is~~ of two kinds, called *ſingle* and *double*.

SINGLE POSITION.

Single poſition teacheth to reſolve thoſe queſtions whoſe reſults are proportional to their ſuppoſitions.

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indices; and then out of this root extract a root anſwering to another of the indices, and ſo on to the laſt.

Thus, the fourth root = ſquare root of the ſquare root.

The ſixth root = ſquare root of the cube root, &c.

The proof of all roots is by involution, or caſting out the nines as in multiplication.

The following theorems may ſometimes be found uſeful in extracting the root of a vulgar fraction; $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}$;

or, univerſally, $\left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\left(ab^{n-1}\right)^{\frac{1}{n}}}{b} = \frac{a}{\left(ba^{n-1}\right)^{\frac{1}{n}}}$.

RULE.

4. Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the given number as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, and so on till the whole is finished.

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1. What is the cube root of 53157376?

$$
\begin{array}{rl}
 & \dot{5}3\dot{1}57\dot{3}76(376 \\
 & 27 = 3^3 \\
\hline
3^2 \times 3 = 27) & 261 \ \textit{dividend} \\
\hline
 & 50653 = 37^3 \\
\hline
3^2 \times 3 = 4107) & 25043 \ \textit{second dividend} \\
 & 53157376 \\
\hline
 & 0
\end{array}
$$

2. What is the biquadrate root of 19987173376?

Anʃ. 376

3. Extract

The extracting of the roots of very high powers by this rule will be found a tedious operation, and will be made use of only by those who are not acquainted with other methods.

When the index of the power whose root is to be subtracted is a composite number, the following rule will be serviceable:

Take any two or more indices, whose product is the given index, and extract out of the given number a root answering to one of these indices;

3. Extract the surfolid, or fifth root, of 3076828211 06715625. *Ans.* 3145

4. Extract the square cubed, or sixth root, of 435728381009267809889764416 *Ans.* 27534

5. Find the seventh root of 344877174673075131824921537 94673. *Ans.* 32017

6. Find the eighth root of 1121016281320476236246497942460481. *Ans.* 13527

7. Find the ninth root of 976379602989073960279630298890. *Ans.* 2148.7201

POSITION.

Position is a method of performing such questions as cannot be resolved by the common direct rules~~, and is~~ of two kinds, called *single* and *double*.

SINGLE POSITION.

Single position teacheth to resolve those questions whose results are proportional to their suppositions.

---

indices; and then out of this root extract a root answering to another of the indices, and so on to the last.

Thus, the fourth root $=$ square root of the square root.

The sixth root $=$ square root of the cube root, &c.

The proof of all roots is by involution, or casting out the nines as in multiplication.

The following theorems may sometimes be found useful in extracting the root of a vulgar fraction; $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}$;

or, universally, $$\overline{\frac{a}{b}}\Big|^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\overline{ab^{n-1}}\Big|^{\frac{1}{n}}}{b} = \frac{a}{\overline{ba^{n-1}}\Big|^{\frac{1}{n}}}.$$

RULE.

RULE.*

1. Take any number and perform the same operations with it as are described to be performed in the question.

2. Then say, as the result of the operation is to the position, so is the result in the question to the number required.

EXAMPLES.

1. A's age is double of B's, and B's is triple of C's, and the sum of all their ages is 140: what is each person's age?

Suppose A's age to be 60

$$\textit{Then will B's} = \frac{60}{2} = 30$$

$$\textit{And C's} = \frac{30}{3} = 10$$

100 *sum.*

$$\textit{As } 100 : 60 :: 140 : \frac{140 \times 60}{100} = 84 = \textit{A's age.}$$

$$\textit{Conseq.}\ \frac{84}{2} = 42 = \textit{B's}$$

$$\textit{And}\ \frac{42}{3} = 14 = \textit{C's}$$

140 *Proof.*

2. A certain sum of money is to be divided between 4 persons, in such a manner, that the first shall have $\frac{1}{3}$ of

---

* Such questions properly belong to this rule as require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself, or any parts of itself, a certain proposed number of times. For in this case the reason of the rule is obvious; it being, then, evident, that the results are proportional to the suppositions.

Thus,

of it; the second $\frac{1}{4}$; the third $\frac{1}{6}$; and the fourth the remainder, which is 28 *l.* : what was the sum?

Ans. 112 *l.*

3. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money had 60 *l.* left: what had he at first? *Ans.* 144 *l.*

4. What number is that which being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of itself, the sum shall be 125. *Ans.* 60

5. A person bought a chaise, horse and harness, for 60*l.*; the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness: what did he give for each? *Ans.* 13*l.* 6*s.* 8*d.* *for the horse,* 6*l.* 13*s.* 4*d.* *for the harness, and* 40*l.* *for the chaise.*

6. A vessel has 3 cocks, A, B and C; A can fill it in 1 hour, B in 2, and C in 3: in what time will they all fill it together? *Ans.* $\frac{6}{11}$ *hours.*

DOUBLE POSITION.

Double position teacheth to resolve questions by making two suppositions of false numbers.

RULE.*

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Find

$$\text{Thus,}\ \begin{cases} nx : x :: na : a \\ \dfrac{x}{n} : x :: \dfrac{a}{n} : a \\ \dfrac{x}{n} \pm \dfrac{x}{m}\ \&c. : x :: \dfrac{a}{n} \pm \dfrac{a}{m}\ \&c. : a \text{ and so on.} \end{cases}$$

Note, 1 may be made a constant supposition in all questions; and in most cases it is better than any other number.

* The rule is founded on this supposition, that the first error is to the second, as the difference between the true and first supposed number, is to the difference between the true and second supposed number: when that is not the case, the exact answer to the question cannot be found by this rule.

That the rule is true, according to the supposition, may be thus demonstrated. Let

2. Find how much the results are different from the result in the question.

3. Multiply each of the errors by the contrary supposition, and find the sum and difference of the products.

4. If the errors are alike, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

5. If the errors are unlike, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Note, The errors are said to be alike, when they are both too great or both too little; and unlike, when one is too great and the other too little.

EXAMPLES.

1. A lady bought tabby at 4*s.* a yard, and persian at 2*s.* a yard; the whole number of yards she bought were 8, and the whole price 20*s.*: how many yards had she of each sort?

Let A and B be any two numbers produced from *a* and *b* by similar operations; it is required to find the number from which N is produced by a like operation.

Put $x =$ number required, and let $N - A = r$, and $N - B = s$.

Then, according to the supposition on which the rule is founded, $r : s :: x - a : x - b$, whence, by multiplying means and extremes, $rx - rb = sx - sa$; and by transposition $rx - sx = rb - sa$; and by division $x = \frac{rb - sa}{r - s} =$ number sought.

Again, if r and s be both negative, we shall have $-r : -s :: x - a : x - b$, and therefore $-rx + rb = -sx + sa$; and $rx - sx = rb - sa$; from whence $x = \frac{rb - sa}{r - s}$ as before.

In like manner, if r or s be negative, we shall have, $x = \frac{rb + sa}{r + s}$, by working as before, which is the rule.

Note, it will be often advantageous to make 1 and 0 the suppositions.

Suppose

Suppose 4 yards of tabby, value 16 *s.*
Then she must have 4 yards of persian, value 8

Sum of their values 24
So that the first error is + 4

Again, suppose she had 3 yards of tabby at 12 *s.*
Then she must have 5 yards of persian at 10

Sum of their values 22

So that the second error is + 2

Then $4 - 2 = 2 =$ *difference of the errors.*
Also $4 \times 2 = 8 =$ *product of the first supposition and second error.*
And $3 \times 4 = 12 =$ *product of the second supposition by the first error.*
And $12 - 8 = 4 =$ *their difference.*
Whence $4 \div 2 = 2 =$ *yards of tabby.* } *the ans.*
And $8 - 2 = 6 =$ *yards of persian.* }

2. Two persons, A and B, have both the same income; A saves $\frac{1}{5}$ of his yearly; but B, by spending 50*l.* *per annum* more than A, at the end of 4 years finds himself 100*l.* in debt: what is their income, and what do they spend *per annum?* *Ans. Their income is* 125 *l. per ann. also A spends* 100 *l. and B* 150 *l. per ann.*

3. Two persons, A and B, lay out equal sums of money in trade; A gains 126 *l.* and B loses 87 *l.* and A's money is now double of B's: what did each lay out? *Ans.* 300 *l.*

4. A labourer was hired for 40 days, upon this condition, that he should receive 20 *d.* for every day he wrought, and forfeit 10*d.* for every day he was idle: now he received at last 2*l.* 1*s.* 8*d.*: how many days did he work, and how many was he idle? *Ans. wrought* 30 *days, and was idle* 10.

5. A gentleman has two horſes of conſiderable value, and a ſaddle worth 50 *l.*; now, if the ſaddle be put on the back of the firſt horſe, it will make his value double that of the ſecond; but if it be put on the back of the ſecond, it will make his value triple that of the firſt: what is the value of each horſe?

Anſ. One 30 *l. and the other* 40 *l.*

6. There is a fiſh whoſe head is 9 inches long, and his tail is as long as his head and half as long as his body, and his body is as long as his tail and his head: what is the whole length of the fiſh? *Anſ.* 3 *feet*

OF PERMUTATIONS AND COMBINATIONS.

The combination of quantities, is the ſhewing how often a leſs number of things can be taken out of a greater, and combined together, without conſidering their places, or the order they ſtand in.

This is ſometimes called *election* or *choice*; and here every parcel muſt be different from all the reſt, and no two are to have preciſely the ſame quantities, or things.

The permutation of quantities, is the ſhewing how many different ways any given number of things may be changed.

This is alſo called *variation*, *alternation*, or *changes*; and the only thing to be regarded here is the order they ſtand in; for no two parcels are to have all their quantities placed in the ſame ſituation.

The compoſition of quantities, is the taking a given number of quantities, out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places; and it differs from combination only, as that admits of but one row of things.

Combinations of the ſame form, are thoſe in which there are the ſame number of quantities, and the ſame repetitions:

titions: thus, *abcc*, *bbad*, *deef*, *&c.* are of the same form; but *abbc*, *abbb*, *aacc*, *&c.* are of different forms.

PROB. I.

To find the number of permutations, or changes, that can be made of any given number of things all different from each other.

RULE.*

Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

EXAMPLES.

1. How many changes may be rung on 6 bells?

$$\begin{array}{r} 1 \\ 2 \\ \hline 2 \\ 3 \\ \hline 6 \\ 4 \\ \hline 24 \\ 5 \\ \hline 120 \\ 6 \\ \hline 720 \end{array}$$

Or, $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ *the answer.*

2. For

* The reason of the rule may be shewn thus: any one thing *a* is capable only of one position, as *a*.

Any

2. For how many days can 7 persons be placed in a different position at dinner? *Ans.* 5040 *days*

3. How many changes may be rung on 12 bells, and how long would they take in ringing, supposing 10 changes to be rung in 1 minute, and the year to consist of 365 days, 5 hours and 49 minutes? *Ans.* 479001600 *changes, and* 91 *years,* 26 *days,* 22 *ho.* 41 *min.*

4. How many changes may be made of the words in the following verse? *Tot tibi sunt dotes, virgo, quot sydera cœlo.* *Ans.* 40320 *changes.*

PROB. 2.

Any number of different things being given; to find how many changes can be made out of them, by taking any given number of quantities at a time.

RULE.*

Take a series of numbers, beginning at the number of things given, and decreasing by 1 to the number of quantities

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Any two things *a* and *b*, are only capable of two variations; as *ab*, *ba*; whose number is expressed by 1×2.

If there be 3 things *a*, *b* and *c*; then any two of them, leaving out the 3d, will have 1×2 variations; and consequently, when the 3 are taken in, there will be $1 \times 2 \times 3$ variations.

In the same manner, when there are 4 things, every three, leaving out the 4th, will have $1 \times 2 \times 3$ variations. Then, taking in successively the 4 left out, there will be $1 \times 2 \times 3 \times 4$ variations. And so on as far as you please.

* This rule expressed in terms, is as follows: $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$ &c. to n terms; where m = number of things given, and n = quantities to be taken at a time.

In order to demonstrate the rule, it will be necessary to premise the following

LEMMA.

The number of changes of m things, taken n at a time, is equal to m changes of $m-1$ things taken $n-1$ at a time.

Demon. Let any 5 quantities *a b c d e* be given.

First,

quantities to be taken at a time, and the product of all the terms will be the answer required.

EXAMPLES.

1. How many changes may be rung with 3 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \end{array}$$

Or, $8 \times 7 \times 6$ (= 3 *terms*) = 336 *the answer*.

2. How many words can be made with 5 letters of the alphabet, admitting that a number of consonants may make a word? *Ans.* 5100480

First, leave out the *a*, and let v = number of all the variations of every two, *bc*, *bd* &c. that can be taken out of the 4 remaining quantities *b c d e*.

Now, let *a* be put in the first place of each of them, *a b c*, *a b d*, &c. and the number of changes will still remain the same; that is, v = number of variations of every 3 out of the 5, *a b c d e*, when *a* is first.

In like manner, if *b*, *c*, *d*, *e* be successively left out, the number of variations of all the two's will also = v; and putting *b*, *c*, *d*, *e* respectively in the first place, to make 3 quantities out of 5, there will still be v variations as before.

But these are all the variations that can happen of 3 things out of 5, when *a*, *b*, *c*, *d*, *e* are successively put first; and therefore the sum of all these is the sum of all the changes of 3 things out of 5.

But the sum of these is so many times v as is the number of things; that is $5v$, or mv, = all the changes of 3 things out of 5. And the same way of reasoning may be applied to any numbers whatever.

Demon. of the rule. Let any 7 things *a b c d e f g* be given, and let 3 be the number of quantities to be taken.

Then $m = 7$ and $n = 3$.

Now, it is evident, that the number of changes that can be made by taking 1 by 1 out of 5 things will be 5, which let = v.

 Then,

PROB. 3.

Any number of things being given; whereof there are several given things of one sort, several of another, &c. To find how many changes can be made out of them all.

RULE.*

1. Take the series $1 \times 2 \times 3 \times 4$ *&c.* up to the number of things given, and find the product of all the terms.

Then, by the lemma, when $m = 6$ and $n = 2$, the number of changes will $= mv = 6 \times 5$; which let $= v$ a second time.

Again, by the lemma, when $m = 7$ and $n = 3$, the number of changes $= mv = 7 \times 6 \times 5$; that is $mv = m \times \overline{m-1} \times \overline{m-2}$, continued to 3, or n terms. And the same may be shewn for any other numbers.

* This rule is expressed in terms thus: $\dfrac{1 \times 2 \times 3 \times 4 \times 5 \text{ \&c. to } m}{1 \times 2 \times 3 \text{\&c. to } p \times 1 \times 2 \times 3 \text{\&c. to } q}$ &c.; where $m =$ number of things given, $p =$ number of things of the first sort, $q =$ number of things of the second sort, &c.

The demonstration may be shewn as follows:

Any 2 quantities, $a\ b$, both different, admit of 2 changes; but if the quantities are the same, or $a\ b$ becomes $a\ a$, there will be only one alternation; which may be expressed by $\dfrac{1 \times 2}{1 \times 2} = 1$.

Any three quantities, $a\ b\ c$, all different from each other, afford 6 variations; but if the quantities are all alike, or $a\ b\ c$ becomes $a\ a\ a$, then the 6 variations will be reduced to 1; which may be expressed by $\dfrac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. Again, if two of the quantities only are alike, or $a\ b\ c$ becomes $a\ a\ c$; then the 6 variations will be reduced to these 3, $a\ a\ c$, $c\ a\ a$, and $a\ c\ a$; which may be expressed by $\dfrac{1 \times 2 \times 3}{1 \times 2} = 3$.

Any four quantities, $a\ b\ c\ d$, all different from each other, will admit of 24 variations; but if the quantities are the same, or $a\ b\ c\ d$ becomes $a\ a\ a\ a$, the number of variations will be reduced to one; which is $= \dfrac{1 \times 2 \times 3 \times 4}{1 \times 2 \times 3 \times 4} = 1$. Again, if three of the quantities

only

2. Take the ſeries $1 \times 2 \times 3 \times 4$ &c. up to the number of given things of the firſt ſort, and the ſeries $1 \times 2 \times 3 \times 4$ &c. up to the number of given things of the ſecond ſort, &c.

3. Divide the product of all the terms of the firſt ſeries by the joint product of all the terms of the remaining ones, and the quotient will be the anſwer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Bacchanalia*?

$$1 \times 2 (= \textit{number of } c\textit{'s}) = 2$$

$$1 \times 2 \times 3 \times 4 (= \textit{number of } a\textit{'s}) = 24$$

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11$$
$$(= \textit{number of letters in the word}) = 39916800$$

$2 \times 24 = 48)39916800(831600$ *the anſwer.*

151
76
288

2. How many different numbers can be made of the following figures, 1220005555? *Anſ.* 12600

---

only are the ſame, or $a\,b\,c\,d$ becomes $a\,a\,a\,b$, the number of variations will be reduced to theſe 4, $a\,a\,a\,b$, $a\,a\,b\,a$, $a\,b\,a\,a$, and $b\,a\,a\,a$; which is $= \dfrac{1 \times 2 \times 3 \times 4}{1 \times 2 \times 3} = 4$. And thus it may be ſhewn that if two of the quantities are alike, or the 4 quantities be $a\,a\,b\,c$, the number of variations will be reduced to twelve; which may be expreſſed by $\dfrac{1 \times 2 \times 3 \times 4}{1 \times 2} = 12$.

And by reaſoning in the ſame manner, it will appear that the number of changes which can be made of the quantities $a\,b\,b\,c\,c\,c$ is equal to 60; which may be expreſſed by $\dfrac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{1 \times 2 \times 1 \times 2 \times 3}$ $= 60$; and ſo of any other quantities whatever.

 3. What

3. What is the variety in the ſucceſſion of the following muſical notes, *fa, fa, fa, ſol, ſol, la, mi, fa?*

Anſ. 3360

PROB. 4.

To find the changes of any given number of things, taken a given number at a time; in which there are ſeveral given things of one ſort, ſeveral of another, &c.

RULE.*

1. Find all the different forms of combination of all the given things, taken as many at a time as in the queſtion.

2. Find the number of changes in any form, and multiply it by the number of combinations in that form.

3. Do the ſame for every diſtinct form; and the ſum of all the products will give the whole number of changes required.

---

* The reaſon of this rule is plain from what has been ſhewn before, and the nature of the problem.

A rule for finding the number of forms.

1. Place the things ſo that the greateſt indices may be firſt, and the reſt in order.

2. Begin with the firſt letter, and join it to the ſecond, third, fourth, &c. to the laſt.

3. Then take the ſecond letter, and join it to the third, fourth, &c. to the laſt; and ſo on till they are all done, always remembring to reject ſuch combinations as have occurred before; and this will give the combinations of all the two's.

4. Join the firſt letter to every one of the two's, and the ſecond, third, &c. as before; and it will give the combinations of all the three's.

5. Proceed in the ſame manner to get the combinations of all the fours, &c. and you will at laſt get all the ſeveral forms of combination, and the number in each form.

EXAM-

EXAMPLES.

1. How many alternations, or changes, can be made of every 4 letters out of these 8; *aaabbbcc*?

No. of forms.	*No. of changes.*
a^3b, a^3c, b^3a, b^3c	4
a^2b^2, a^2c^2, b^2c^2	6
a^2bc, b^2ac, c^2ab	12

$$\textit{Therefore}\begin{cases}4 \times 4 = 16\\ 3 \times 6 = 18\\ 3 \times 12 = 36\end{cases}$$

70 = *number of changes required.*

2. How many changes can be made of every 8 letters out of these 10; *aaaabbccde*? *Ans.* 22260

3. How many different numbers can be made out of 1 unit, 2 two's, 3 three's, 4 four's, and 5 five's; taken 5 at a time? *Ans.* 2111.

PROB. 5.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.*

1. Take the series 1, 2, 3, 4 &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take

* This rule, expressed algebraically, is, $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c. to n terms; where m is the number of given quantities, and n those to be taken at a time.

Demon. of the rule. 1. Let the number of things to be taken at a time be 2, and the things to be combined $= m$.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

EXAM-

Now, when m, or the number of things to be combined, is only two, as a and b, it is evident that there can be only one combination, as ab; but if m be increased by one, or the letters to be combined be 3, as a, b, c, then it is plain that the number of combinations will be increased by 2, since with each of the former letters a and b the new letter c may be joined. It is evident, therefore, that the whole number of combinations, in this case, will be truly expressed by $1 + 2$.

Again, if m be increased by one letter more, or the whole number of letters be four, as a, b, c, d; then it will appear that the whole number of combinations must be increased by 3, since with each of the preceding letters the new letter c may be combined. The combinations, therefore, in this case, will be truly expressed by $1 + 2 + 3$.

In the same manner, it may be shewn, that the whole number of combinations of 2, in 5 things, will be $1 + 2 + 3 + 4$; of 2, in 6 things, $1+2+3+4+5$; and of 2, in 7, $1+2+3+4+5+6$, &c.

Whence, universally, the number of combinations of m things, taken 2 by 2, is $= 1 + 2 + 3 + 4 + 5 + 6$ &c. to $\overline{m-1}$ terms.

But the sum of this series is $= \frac{m}{1} + \frac{m-1}{2}$; which is the same as the rule.

2. Let now the number of quantities in each combination be supposed to be three.

Then it is plain, that, when $m=3$, or the things to be combined are a, b, c, there can be only one combination; but if m be increased by 1, or the things to be combined are 4, as a, b, c, d, then will the number of combinations be increased by 3; since 3 is the number of combinations of 2 in all the preceding letters a, b, c, and with each two of these the new letter d may be combined.

The number of combinations, therefore, in this case, is $1 + 3$.

Again, if m be increased by one more, or the number of letters be supposed 5; then the former number of combinations will be increased by 6, that is, by all the combinations of 2 in the 4 preceding letters, a, b, c, d; since, as before, with each two of these the new letter e may be combined.

The

EXAMPLES.

1. How many combinations can be made of 6 letters out of 10?

$1 \times 2 \times 3 \times 4 \times 5 \times 6$ (= *the number to be taken at a time*) = 720

$10 \times 9 \times 8 \times 7 \times 6 \times 5$ (= *same number from* 10) = 151200

720(151200(210 *the answer.*
1440

720
720

0

2. How many combinations can be made of 2 letters out of the 24 letters of the alphabet? *Ans.* 276

3. A general, who had often been successful in war, was asked by his king what reward he should confer upon him for his services; the general only desired a farthing for every file, of 10 men in a file, which he could make with a body of 100 men; what is the amount in pounds sterling? *Ans.* 18031572350 *l.* 9 *s.* 2 *d.*

The number of combinations, therefore, in this case, is $1 + 3 + 6$.

Whence, universally, the number of combinations of m things, taken 3 by 3 is $1 + 3 + 6 + 10$ &c. to $m-2$ terms.

But the sum of this series is $= \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$; which is the same as the rule.

And the same thing will hold let the number of things to be taken at a time be what they will; therefore the number of combinations of m things, taken n at a time, will $= \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c. to n terms. *Q. E. D.*

PROB. 6.

To find the number of combinations of any given number of things, by taking any given number at a time; in which there are several things of one sort, several of another, &c.

RULE.

1. Find, by trial, the number of different forms which the things to be taken at a time will admit of, and the number of combinations there are in each.

2. Add all the combinations, thus found, together, and the sum will be the number required.

EXAMPLES.

1. Let the things proposed be $a\,a\,a\,b\,b\,c$; it is required to find the number of combinations made of every 3 of these quantities.

Forms.	*Combinations.*
a^3	1
a^2b, a^2c, b^2a, b^2c	4
abc	1
	6 = *num-*

ber of combinations required.

2. Let $a\,a\,a\,b\,b\,b\,c\,c$ be proposed; it is required to find the number of combinations of these quantities taken 4 at a time. *Ans.* 10.

3. How many combinations are there in $a\,a\,a\,a\,b\,b\,c\,c\,d\,e$, taking 8 at a time? *Ans.* 13

4. How many combinations are there in $a\,a\,a\,a\,a\,b\,b\,b\,b\,b\,c\,c\,c\,c\,d\,d\,d\,d\,e\,e\,e\,e\,f\,f\,f\,g$, taking 10 at a time? *Ans.* 2819.

PROB. 7.

To find the compositions of any number, in an equal number of sets, the things themselves being all different.

RULE.*

Multiply the number of things in every ſet continually together, and the product will be the anſwer required.

EXAMPLES.

1. Suppoſe there are 4 companies, in each of which there are 9 men; it is required to find how many ways 9 men may be choſen, one out of each company,

$$\begin{array}{r} 9 \\ \underline{9} \\ 81 \\ \underline{9} \\ 729 \\ \underline{9} \\ 6561. \end{array}$$

Or, $9 \times 9 \times 9 \times 9 = 656$ *the anſwer.*

---

* *Demon.* Suppoſe there are only two ſets; then, it is plain, that, every quantity of the one ſet being combined with every quantity of the other, will make all the compoſitions, of two things in theſe two ſets; and the number of theſe compoſitions is, evidently, the product of the number of quantities in one ſet by that in the other.

Again, ſuppoſe there are three ſets; then the compoſition of two, in any two of the ſets, being combined with every quantity of the third, will make all the compoſitions of 3 in the 3 ſets. That is, the compoſitions of 2, in any two of the ſets, being multiplied by the number of quantities in the remaining ſet, will produce the compoſitions of three in the three ſets; which is, evidently, the continual product of all the three numbers in three ſets. And the ſame manner of reaſoning will hold, let the number of ſets be what it will. *Q. E. D.*

The doctrine of permutations, combinations, *&c.* is of very extenſive uſe in different parts of the mathematics; particularly in the calculation of annuities and chances. The ſubject might have been purſued to a much greater length; but what has been done already will be found ſufficient for moſt of the purpoſes to which things of this nature are applicable.

2. Suppose there are 4 companies; in one of which there are 6 men, in another 8, and in each of the other two, 9; what are the choices, by a composition of 4 men, one out of each company? *Ans.* 3888

3. How many changes are there in throwing 5 dice? *Ans.* 7776

EXCHANGE.

Exchange is the method of bartering the money of one place for that of another, and consists in finding what sum of the money of one country will be equal to any given sum of another, according to a certain given course of exchange.

The course of exchange is such a variable sum of the money of one place, as is proposed to be given for a certain constant sum of that of another.

The par of exchange is that quantity of the money of one country, which is intrinsically equal to a certain quantity of the money of another; and it is one of these that is the constant sum to which the course is compared.

The money in the banks of foreign places is finer than that which is current in those places; and the difference between any sum as it is valued in the one or the other is called the *agio*.

The money made use of in exchange is generally imaginary; and in most places differs very widely from the money in which they keep their accounts. It is also to be observed, that in many places, the money made use of in exchange, and the money which is current, is very different, as well as that of banco and current.

All the operations in exchange may be performed by the rule of three and practice.

ENGLAND.

ENGLAND, WITH HOLLAND, FLANDERS AND GERMANY.

Accounts are kept in these places in guilders, stivers and pennings; or in pounds, shillings and pence as in England.

The money of Holland and Flanders is distinguished by the name of *flemish*, and they exchange by the pound sterling.

8 pennings	make one	grote or penny
2 grotes		stiver
6 stivers		schilling
20 stivers		florin or guilder
$2\frac{1}{2}$ florins		rix-dollar
6 florins		pound flemish.

Exchange from 33*s.* 6*d.* to 36*s.* 6*d.* flem. *per pound* sterling.

Agio from 3 to 6 *per cent.* for current.

To turn current money into banco.

RULE.

As 100 with the agio added to it, is to 100, so is any given sum current to its value banco.

To turn banco money into current.

RULE.

As 100, is to 100 with the agio added to it, so is any given sum banco to its value current.

Note, The exchange is supposed to be made in bank money, and therefore current money must always be turned into banco before the exchange can be made.

EXAM-

EXAMPLES.

1. In 96*l.* 6*s.* 11*d.* ſterling, how many florins, &c. exchange at 34*s.* 3*d.* flemiſh *per pound* ſterling?

		l.	*s.*	*d.*
		96*l.*	6*s.*	11*d.*
10*s.*	*is* $\frac{1}{2}$	48 -	3 -	$5\frac{1}{2}$
3*s.* 4*d.*	*is* $\frac{1}{6}$	16 -	1 -	$1\frac{3}{4}$
10*d.*	*is* $\frac{1}{4}$	4 -	0 -	$3\frac{1}{2}$
1*d.*	*is* $\frac{1}{10}$	- -	8 -	- $\frac{1}{4}$
		164 -	19 -	10
				6
		989 -	19 -	-

Anſ. 989 *flor.* 19 *ſt.*

2. In 612*l.* 14*s.* $9\frac{1}{2}$*d.* ſterling, how many Dutch rix-dollars, exchange 35*s.* 4*d.* $\frac{7}{8}$ flem *per l.* ſterling?
Anſ. 2603 *rix-dol.* 18 *ſt.* 1 *gr.* 5 *pen.*

3. In 3758 *flor.* 15 *ſt.* current, agio $5\frac{5}{8}$ *per cent.* how many pounds ſterling, exchange at 35 *s.* 11 *d.*?
Anſ. 330*l.* 5*s.* $2\frac{1}{4}$*d.*

4. In 456*l.* 8*s.* ſterling, how many rix-dollars current, agio $4\frac{5}{8}$, exchange 36 *s.* $1\frac{1}{2}$*d.*?
Anſ. 2069 *rix-dol.* 2 *flor.* 10 *ſt.*

5. In 2714 *guil.* 15 *ſt.* how many pounds ſterling; exchange at 35*s.* 6*d.* flemiſh *per* pound ſterling?
Anſ. 254*l.* 18*s.* $1\frac{1}{4}$*d.*

6. In 290*l.* 11*s.* 10*d.* ſterling, how many pounds flemiſh; exchange at 33*s.* 10*d.* flem. *per* pound ſterling; and agio at $4\frac{1}{2}$ *per cent*? *Anſ.* 513*l.* 14*s.* $1\frac{1}{4}$*d.*

7. In 805*l.* 15*s.* flemiſh, how many pounds ſterling; the agio being 4 *per cent*, and exchange 34*s.* 6*d.* flem. *per* pound ſterling? *Anſ.* 449*l.* 2*s.* $8\frac{1}{2}$*d.*

8. The

8. The courſe of exchange, this day March 29, 1779, between London and Amſterdam is 34 *s.* 3 *d.* at 2 ½ uſance, what ought Amſterdam to give at ſight, ſuppoſing the intereſt of money to be 4 *per cent.*?

Anſ. 33 *s.* 11¾ *d.*

HAMBRO.

They keep their accounts at Hambro in marks and ſols lub, and exchange by the pound ſterling as in Holland:

2 deniers gros	make one	ſol lubs
6 ſol lubs		ſol gros
16 ſol lubs		mark
2 marks		drittle, or Hambro dollar
3 marks		rix-dollar
7½ marks		livre gros, or pound flem.

Exchange from 32*s.* to 35*s.* flem *per l.* ſterling.

Agio from 18 to 20 *per cent.* for current, and from 30 to 35 *per cent.* for ſlight.

EXAMPLES.

1. In 3459 *mar.* 10 *ſol. l.* banco, how many pounds ſterling, exchange 36 *ſol. g.* 1 *den.* *per* pound ſterling?

36 *ſol g.* 1 *den.* : 1 *l.* :: 3459 *mar.* 10 *ſol gr.*

```
 12                        16
----                     -----
433                      20754
                         3460
                         -----
                         55354
                             2
                         -----
                  433)110708(255 l.
                       2410
                        1458
                         293
                          20
```

Anſ. 255 *l.* 13 *s.* 6 ¼ *d.* —— &c.

2. In 255*l.* 13*s.* 6¼*d.* sterling, how many marks, &c. exchange 36 *sol gros.* 1 *den.* *per* pound sterling?

Ans. 3459 *mar.* 10 *sol l.*

3. In 536*l.* sterling how many marks; exchange at 36*s.* 4*d.* flemish *per* pound sterling?

Ans. 7303 *marks.*

4. In 127*l.* 3*s.* 4*d.* sterling, how many Hambro marks, exchange at 32⅓ *sol gros per* pound sterling?

Ans. 1541 *mar.* 14⅓ *sol lubs*

5. In 3065 *rix-doll.* 23 *sol. lubs.* how many pounds sterling, exchange at 32 *sol gros*, 8 *den. per* pound sterling?

Ans. 750*l.* 14*s.* 7*d.*

6. In 585 *rix-doll.* 1 *sol gros*, slight money, agio $4\frac{9}{16}$ *per cent.* exchange 35 *sol gros*, 8½ *den.* how many pounds sterling?

Ans. 125*l.* 7*s.* 4*d.*

7. In 934*l.* 1*s.* 2¼*d.* sterling, how many rix-dollars, &c. current, exchange at 33 *sol gros*, 9¼ *den.* agio 118½

Ans. 4672 *rix-doll.* 22 *sol lubs*

8. In 1075 *marks*, 14 *sol lubs* current, agio 8⅜ *per cent.* and 384 *dol.* 2 *sol gros* slight, agio 4⅞ *per cent.* exchange 35 *sol gros*, 7 *den.* how many pounds sterling?

Ans. 129*l.* 6*s.* 6¼*d.*

FRANCE.

Accounts are kept in France in livres, sols and deniers, and they exchange by the crown tournois.

12 deniers	make one	sol
20 sols	make one	livre
3 livres	make one	ecu, or crown tournois
10 livres	make one	pistole
24 livres	make one	louis d'or, or guinea

Exchange from 30*d.* to 32*d.* sterling *per* ecu.

EXAM-

EXAMPLES.

1. Reduce 3989 *liv.* 13*s.* 9*d.* into pounds sterling, exchange 31¼*d.* *per* ecu.

	liv.	*s.*	*d.*
	3)3989	13	9
	1329	17	11
d.			
30 *is* ⅛	166	4	8¾
1 *is* $\frac{1}{30}$	5	10	9¾
¼ *is* ¼	1	7	8¼
	173*l.*	3*s.*	2¾*d.* *the answer.*

2. In 471*l.* 17*s.* 4½*d.* sterling, how many livres tournois, exchange at 31½*d.* sterling *per ecu?*
Ans. 10785 *liv.* 11 *sols.* 10 *den.*

3. In 771*l.* 17*s.* 6*d.* sterling, how many French pistoles, exchange 30⅞ *d. per ecu?* *Ans.* 1800

4. What comes 732 *liv.* 13*s.* 11*d.* to in London, at 57½*d.* *per* crown at Bourdeaux? *Ans.* 58*l.* 10*s.* 3¼*d.*

SPAIN.

Accounts are kept in Spain in piastres, rials and marvadies, and they exchange by the piastre or piso.

4 marvadies vellon, or 2⅛ marvadies of plate	make one	quarta
8½ quartas, or 34 marvadies vellon	make one	rial vellon
16 quartas, or 34 marvadies of plate	make one	rial of plate
8 rials of plate	make one	(or dollar piso, piastre, piece of 8
5 piastres	make one	Spanish pistole
2 pistoles	make one	doubloon

Exchange from 38*d.* to 42*d.* sterling *per* piso.

EXAM-

EXAMPLES.

1. In 9764 *rials of plate*, how many pounds sterling, exchange at $41\frac{7}{8}$*d. per piso*?

8)9764 *rials plate*

		1220	10	
d.				
40	*is* $\frac{1}{6}$	203	8	4
1	*is* $\frac{1}{40}$	5	1	$8\frac{1}{2}$
$\frac{4}{8}$	*is* $\frac{1}{2}$	2	10	$10\frac{1}{4}$
$\frac{2}{8}$	*is* $\frac{1}{2}$	1	5	5
$\frac{1}{8}$	*is* $\frac{1}{2}$		12	$8\frac{1}{2}$
		212*l.*	19*s.*	$0\frac{1}{4}$ *the answer.*

2. In 8756 *rials vellon*, how many rials of plate?
Ans. 4651 *rials plate*, 10 *qu.*

3. In 4651 *rials of plate*, 10 *q.* how many rials vellon?
Ans 8756

4. In 89641 *quartas*, how many pounds sterling, exchange at $39\frac{1}{2}$*d. per* piastre? *Ans.* 115*l.* 5*s.* $2\frac{3}{4}$*d.*

5. Reduce 7869 *rials vellon*, 19 *mar.* into pounds sterling, exchange $41\frac{1}{2}$*d.* sterling *per piso*.
Ans. 90*l.* 7*s.* $3\frac{1}{4}$*d.*

6. In 89*l.* 2*s.* $11\frac{1}{2}$*d.* sterling, how many rials of plate, &c. exchange at $40\frac{1}{8}$*d. per* piece of eight?
Ans. 4265 *rials plate*, 12 *q.*

7. In 2561 *pisos*, 5 *rials plate*. 3 *q.* how many pounds sterling, exchange $41\frac{1}{2}$*d.* *Ans.* 442*l.* 19*s.* $0\frac{1}{4}$*d.*

8. Bought goods in Spain to the value of 547268 *quartas*, exchange $40\frac{7}{8}$*d.* sterling, how many pounds sterling must I sell them for in England to gain 20 *per cent*? *Ans.* 873*l.* 16*s.* $2\frac{1}{2}$*d.*

PORTUGAL.

Accounts are kept in Portugal in reas and milreas, and the exchange is by the milrea.

400

400 reas	make one	crusadoe
1000 reas, or $2\frac{1}{2}$ crusadoes	make one	milrea

Exchange from 60 *d.* to 67 *d. per* milrea.

EXAMPLES.

1. In 669 *mil.* 72 *reas*, how many pounds sterling, exchange at 5*s.* 7*d.*?

	669 *milreas.*		
s.			
5 *is* $\frac{1}{4}$	167	5	
6 *is* $\frac{1}{10}$	16	14	6
1 *is* $\frac{1}{6}$	2	15	9
72 *reas* =	-	-	$4\frac{3}{4}$
	186*l.*	15*s.*	$7\frac{1}{4}$*d.* *the answer.*

2. In 569*l.* 17*s.* 10*d.* sterling, how many milreas exchange at 5*s.* 6*d.* sterling *per* milrea? *Ans.* 2072 *milreas*, 333 *reas*

3. In 754*l.* 18*s.* 6*d.* sterling, how many crusadoes, exchange $64\frac{1}{2}$? *Ans.* $7022\frac{1}{2}$ *cru.*

4. In 2729 *crusadoes*, 372 *reas*, how much sterling, exchange at 62*d.* *Ans.* 282*l.* 1*s.* 10*d.*

VENICE.

They keep their accounts at Leghorn in dollars, soldi and denari, and exchange by the ducat and piastre.

12 denari	make one	soldo
20 soldi	make one	lira, or piastre of Leghorn
$5\frac{1}{6}$ soldi	make one	grosso
24 grossi	make one	ducat.

Exchange from 52*d.* to 54*d. per* ducat, and from 45*d.* to 54*d. per* piastre.

Agio 20 *per cent.*

EXAM-

EXAMPLES.

1. In 7456 *pias.* 9 *sol.* 6 *den.* lire money, how many pounds sterling, exchange being at $49\frac{7}{8}$*d.* *per* piastre?

d.		7456 *pias.*	9 *s.*	6 *d.*
40 *is*	$\frac{1}{6}$	1242	- 14	- 11
8 *is*	$\frac{1}{5}$	248	- 10	- $11\frac{3}{4}$
1 *is*	$\frac{1}{8}$	31	- 1	- $4\frac{1}{4}$
$\frac{4}{8}$ *is*	$\frac{1}{2}$	15	- 10	- 8
$\frac{2}{8}$ *is*	$\frac{1}{2}$	7	- 15	- 4
$\frac{1}{8}$ *is*	$\frac{1}{2}$	3	- 17	- 8
		1549*l.*	- 10*s.*	- 11*d.* *the answer*

2. In 278*l.* 17*s.* 9*d.* sterling, how many piastres of Leghorn, exchange at $47\frac{3}{8}$*d.* *per* piastre?

Ans. 1412 *pias.* 16 *sol.* 8 *den.*

3. Reduce 1549 *duc.* 18 *sol.* 1 *den.* bank money of Venice, into sterling money, exchange at $47\frac{3}{4}$*d.* sterling *per* ducat. *Ans.* 290*l.* 6*s.* $2\frac{1}{4}$*d.*

4. In 4789 *duc.* 19 *sol.* 3 *den.* current money, how many pounds sterling, exchange at 4*s.* 1*d.* *per* ducat banco, and agio 20 *per cent.*? *Ans.* 814*l.* 16*s.* 5*d.*

5. In 415*l.* 17*s.* 4*d.* sterling, how many ducats, &c. current, agio 20 *per cent.* and exchange at 53*d.* *per* ducat? *Ans.* 2259 *duc.* 19 *grossi*

6. In 100*l.* sterling, how many piastres of Leghorn, exchange $52\frac{1}{2}$*d.* *per* ducat? *Ans.* 2834 *pias.* 5 *sol.* 8 *den.*

RUS-

RUSSIA.

They keep their accounts at Petersburg in rubles and copecs, and exchange by the ruble.

3 copecs	make one	altine
10 copecs		grivena
25 copecs		polpolitin
2 polpolitons		politin
2 politins		ruble
2 rubles		ducat.

Russia exchanges with London by way of Hambro or Amsterdam, at the rate of 48 to 50 stivers *per* ruble; and sometimes directly to London from 4*s.* to 5*s.* *per* ruble.

EXAMPLES.

1. In 2634 *rub.* 58 *cop.* how many pounds sterling, exchange at 4*s.* 8*d.* sterling *per* ruble?

s.	2634 *rub.*		
4 *is* $\frac{1}{5}$	526	- 16	
6 *is* $\frac{1}{8}$	65	- 17	
2 *is* $\frac{1}{3}$	21	- 19	
58 *cop.* =		2	- 8$\frac{1}{4}$

614*l.* - 14*s.* - 8$\frac{1}{4}$ *d.* *the answer.*

2. In 674*l.* 17*s.* 6*d.* sterling, how many rubles, exchange 49 stivers *per* ruble, and 33*s.* 9$\frac{1}{2}$*d.* flemish *per* pound sterling? *Ans.* 2792 *rub.* 4 *gr.* 6 *cop.*

3. A merchant at London remits to his correspondent at Petersbourg 471*l.* 17*s.* 4*d.* *ster.* exchange 34*s.* 9*d.* flemish *per* pound *ster.* for Amsterdam, and the exchange from thence at 50 stivers *per* ruble, how many rubles must the correspondent receive?

Ans. 1967 *rub.* 68 *cop.*

4. In

4. Received from Archangel *per* bill of exchange 4675 *rub.* 46 *cop.* exchange 122 copecs *per* rix-dollar of 50 ſtivers, and 34*s.* 7*d.* flemiſh *per* pound ſterling: how much ſterling is the ſum? *Anſ.* 923*l.* 9*s.* $1\frac{1}{4}$*d.*

5. In 4675 *rub.* 46 *cop.* how many pounds ſterling? exchange 122 copecs *per* rix-dollar current, agio three *per cent.* and 34*s.* 7*d.* flemiſh *per* pound ſterling.

Anſ. 896*l.* 11*s.* $2\frac{1}{4}$*d.*

POLAND AND PRUSSIA.

They keep their accounts at Dantzig in florins, gros, and penins, and exchange by the gros.

18 penins	make one	gros
18 gros	make one	oort
30 gros	make one	florin or poliſh guilder
3 florins	make one	rix-dollar
2 rix-dollars	make one	gold ducat

Exchange is made with Poland and Pruſſia by way of Holland, the exchange being from 240 to 295 groſſi *per* pound flemiſh.

EXAMPLES.

1. In 478*l.* 14*s.* 9*d.* ſterling how many Pruſſia florins, &c. exchange 255 groſſi *per* pound flemiſh, and 33*s.* 6*d.* flemiſh *per* pound ſterling?

		l.	*s.*	*d.*
		478*l.*	14*s.*	9*d.*
10*s.*	is $\frac{1}{2}$	239 -	7 -	$4\frac{1}{2}$
3*s.* 4*d.*	is $\frac{1}{6}$	79 -	15 -	$9\frac{1}{2}$
2*d.*	is $\frac{1}{20}$	3 -	19 -	9
		801*l.* -	17*s.* -	8*d.*
				$8\frac{1}{2}$ = 255 *groſſi.*
		6415 -	1 -	4
		400 -	18 -	10
		6816 *florins, the anſwer.*		

2. In

2. In 6949 *flor.* 14 *g.* 2 *pen.* Poliſh, how many pounds ſterling, exchange 260 ½ Poliſh groſſi *per* pound flemiſh, and 34*s.* 8*d.* flemiſh *per* pound ſterling?

Anſ. 461*l.* 14*s.* 5¾*d.*

3. In 875*l.* 14*s.* 8*d.* ſterling, how many rix-dollars, &c. Poliſh, exchange 290 groſſi Poliſh *per* pound flemiſh, and 34*s.* 4*d.* flemiſh *per* pound ſterling?

Anſ. 4844 *rix doll.* 9 *g.* 1 *pen.*

4. In 674*l.* 18*s.* 4*d.* ſterling, how many Poliſh guilders, &c. exchange 274 Poliſh groſſi *per* pound flemiſh, and 35*s.* 6*d.* flemiſh *per* pound ſterling?

Anſ. 10941 *guil.* 15 *g.* 12 *pen.*

5. In 546*l.* 17*s.* 8*d.* ſterling, how many gold ducats, exchange 295 groſſi *per* pound flemiſh, and 33*s.* 10*d.* flemiſh *per* pound ſterling?

Anſ. 1516 *duc.* 37 *g.* 7 *pen.*

SWEDEN.

They keep their accounts at Stockholm in copper dollars and orts, or in ſilver dollars, and exchange by the copper dollar.

8 penins	make one	runſtychen
3 runſtychens	make one	ſtiver, or whitton
8 ſtivers	make one	marc
10 ſtivers and 2 runſtychens, or 32 runſtychens	make one	copper dollar
3 copper dollars and 32 ſtiv. or 96 runſtychens, or 4 marc.	make one	ſilver dollar
24 marcs	make one	copper rix dollar.

The exchange here is ſubject to great variations, but is uſually from 46 to 50 copper dollars *per* pound ſterling,

EXAMPLES.

1. In 146*l.* 17*s.* 6*d.* sterling, how many copper dollars, exchange 48 ½ copper dollars *per* pound sterling?

```
146 l. 17 s. 6 d.
             6
881 - 5 - -
          8
7050 - - - -
  73 - 8 - 9
7123 - 8 - 9
           8
      5)70 - -
        14
```

Ans. 7123 *cop. doll.* 14 *run.*

2. In 546*l.* 19*s.* 6½*d.* sterling, how many silver dollars, exchange 49 ½ copper dollars *per* pound sterling?
Ans. 9025 *sil. doll.* 11 *run.* 5 *pen.*

3. In 674*l.* 11*s.* 6*d.* sterling, how many marcs, &c. exchange 48 copper dollars *per* pound sterling?
Ans. 43172 *marcs*, 6 *st.* 9 *pen.*

4. In 11676 *silver doll.* 18 *run.* 7 *pen.* how many pounds sterling, exchange 49 copper dollars *per* pound sterling? *Ans.* 714*l.* 17*s.* 4½*d.*

5. In 111*l.* 5*s.* 2½*d.* sterling, how many Danish rix-dollars, exchange 35*s.* 7*d.* flemish *per* pound sterling, 106 Amsterdam rix-dollars current for 100 Danish rix-dollars, and agio 3 ¾? *Ans.* 465 *dan. rix-doll.*

IRELAND.

Accounts are kept in Ireland in pounds, ſhillings and pence Iriſh, divided as in England; but having no coins of their own, they are ſupplied by the different countries with which they traffic.

The courſe of exchange between England and Ireland is from 5 to 12 *per cent.* according to the balance of trade.

EXAMPLES.

1. London remits to Ireland 787*l.* 15*s.* ſterling; how much Iriſh muſt London be credited, exchange at 10 $\frac{1}{2}$ *per cent.*?

```
  787 l. 15 s.
         10
 ------------
  7877 - 10
         10
 ------------
 78775 - —
   393 - 17 - 6
 ----------------
 791.68 - 17 - 6
     20
   ----
  13.77
     12
   ----
   3.30
      4
   ----
   1.20
```

Anſ. 791 *l.* 13 *s.* $3\frac{1}{4}$ *d.*

2. Ireland remits to London 879*l.* 6*s.* 6*d.* Iriſh; how much ſterling muſt Ireland be credited, exchange 11 $\frac{5}{8}$ *per cent*? *Anſ.* 787*l.* 15*s.* *ſter.*

3. London remits to Ireland, 540 *l.* 10 *s.* sterling: how much Irish must London be credited, exchange 12 *per* cent? *Ans.* 605 *l.* 7 *s.* 2*d.*

AMERICA AND THE WEST INDIES.

In the American colonies, and the West Indies, accounts are kept in pounds, shillings and pence as in England, and their money is called currency.

The scarcity of cash obliges them to substitute a paper currency for carrying on their trade; which being subject to many casualties, suffers a very great discount for sterling in the purchase of bills of exchange.

EXAMPLES.

1. Philadelphia is indebted to London 1575*l.* 14*s.* 9*d.* currency; what sterling may London reckon to be remitted when the exchange is 35 *per cent.*?

1575 *l.*	14 *s.*	9 *d.*
		4
6302 -	19 -	-
		5
3)31514 -	15 -	-
9)10504 -	18 -	4
1167 *l.*	4 *s.*	3 *d.* *the answer.*

2. London consigns to Virginia goods amounting to 578*l.* 19*s.* 6*d.* which are sold for 847*l.* 15*s.* 6*d.* currency, what sterling ought the factor to remit, deducting 5 *per cent.* for commission and charges, and what does London gain *per cent.* upon the adventure, supposing the exchange at 30 *per cent*? *Ans.* 8*l.* 9*s.* 3¼*d.*

3. Vir-

3. Virginia is indebted to London 575*l.* 19*s.* 6*d.* sterling: with how much currency will London be credited at Virginia, when the exchange is $33\frac{1}{3}$ *per cent*?

Ans. 767*l.* 19*s.* 4*d.*

ARBITRATION OF EXCHANGES.

As the price of exchange, in every place, is continually varying, the arbitration is nothing more than a method of finding such a rate of exchange between any two places, as shall be in proportion with the rates assigned between each of them and a third place. And it is by comparing the par of exchange, thus found, with the present course of exchange, that a person can judge which way to remit or draw to the most advantage, and what the advantage shall be.

All questions in this rule may be performed by one or more operations in the rule of three. *

---

* Any number of operations in the rule of three may be reduced into a single one, thus:

Multiply the consequents of all the proportions into one another continually for a dividend; and all the antecedents, except the first, for a divisor; then will the quotient, arising from this division, be the answer required.

Example. The exchange between London and Amsterdam is 1*l.* sterling for 38*s.* flemish; betwixt Amsterdam and Francfort it is 6*s.* flemish for 65 cruitzers; and between Francfort and Paris it is 56 cruitzers for a crown: what is the exchange between London and Paris?

Lond. Amst. Frank. Paris.
1*l.* = 38*s.*
6*s.* = 66 *cru.*
54 *cru.* = 1 *cr.*

$$\frac{1 \times 38 \times 66}{6 \times 54 \times 1} = \frac{2508}{324} = 7\tfrac{20}{27} \text{ crowns, the answer.}$$

Compound arbitration of exchanges is only a continuation of several statings in simple arbitration.

EXAMPLES.

1. If the exchange between London and Amsterdam be 33*s.* 9*d.* *per* pound sterling, and the exchange between London and Paris be 32*d.* *per* ecu: what is the par of arbitration between Amsterdam and Paris?

```
240d. : 33s. 9d. : : 32
             12
           -----
            405
             32
           -----
            810
           1215
           -----
     24,0)1296,0(54
           120
           -----
            96
            96
            --
```

Ans. 54*d. flem. per ecu.*

2. Amsterdam changes on London at 34*s.* 4*d.* *per* pound sterling, and on Lisbon at 52*d.* flemish for 400 reas: how ought the exchange to go between London and Lisbon? *Ans.* 75 $\frac{75}{103}$*d. sterling per milrea.*

3. London exchanges on Amsterdam at 34*s.* 9*d.* *per* pound sterling, and on Lisbon at 5*s.* 5$\frac{5}{8}$*d.* *per* milrea: what is the arbitrated price between Amsterdam and Lisbon? *Ans.* 45 $\frac{39}{64}$ *flem. per crusadoe*

4. London is indebted to Petersburg 5000 *rubles*: now the exchange between Petersburg and England is at 50*d.* *per* ruble; between Petersburg and Holland 90*d.* *per* ruble; and between Holland and England 36*s.* 4*d.*: which will be the most advantageous method for London to be drawn upon? *Ans. London will gain* 9*l.* 11*s.* 1$\frac{3}{4}$*d. by making payment by way of Holland.*

5. Amsterdam

5. Amsterdam hath orders to remit a certain sum to Cadiz; at the time of this order Amsterdam can remit to Cadiz at $94\frac{3}{4}$*d.* *per* ducat of 375 marvadies, and London to Cadiz at 38*d.* *per* piastre of 272 marvadies: which will be the most advantageous for Amsterdam to remit directly to Cadiz, or by London, the change between Amsterdam and London being 35 *fl.* 10 *guild.* *per* pound sterling? *Ans.* 18*s.* 8*d.* $\frac{1}{4}$ *per cent. in favour of Amsterdam.*

6. A merchant at London hath 6000 guilders in the bank at Amsterdam, and was offered 22*d.* sterling apiece for them; but not liking the offer, he indorsed a bill for the whole to his factor at Paris; who brought the money to France, by exchanging at 55*d.* flemish *per* crown. He allowed the factor $\frac{1}{2}$ *per cent.* commission for his trouble, and then drew upon him for the whole, exchange at 32*d.* *per* ecu: how much was this better than the offer at 22 *d.* *per* guilder?

Ans. 28*l.* 18*s.* 2*d.*

MISCELLANEOUS QUESTIONS.

1. What part of 3*d.* is a third part of 2*d*? *Ans.* $\frac{2}{9}$

2. A has by him 1 $\frac{1}{2}$ *cwt.* of tea, the prime cost of which was 96*l.* Now, granting interest to be at 5 *per cent.* it is required to find how he must rate it *per lb.* to B, so that by taking his negotiable note, payable at 3 months, he may clear 20 guineas by the bargain?

Ans. 14*s.* 1 $\frac{13}{56}$ *d.*

3. What annuity is sufficient to pay off 50 millions of pounds in 30 years at 4 *per cent.* compound interest?

Ans. 2891505 *l.*

4. Sold a piece of cloth containing 1000 flemish ells for 850 guineas, and gained upon every yard $\frac{1}{8}$ of the prime cost of an english ell: what did the whole piece stand me in? *Ans.* 771*l.* 17*s.* 10 $\frac{2}{37}$*d.*

5. The hour and minute hand of a clock are exactly together at 12 o'clock; when are they next together?

Ans. 1 *h.* 5 $\frac{5}{11}$ *min.*

6. There is an iſland 73 miles in circumference, and 3 footmen all ſtart together to travel the ſame way about it; A goes 5 miles a day, B 8, and C 10: when will they all come together again? *Anſ.* 73 *days*

7. Sold goods for 60 guineas, and by ſo doing loſt 17 *per cent.* whereas I ought, in dealing, to have cleared 20 *per cent.*: how much were they ſold under their juſt value? *Anſ.* 28*l.* 1*s.* 8 $\frac{20}{83}$*d.*

8. If, by ſelling goods at 2*s.* 3*d.* *per lb.* I clear *cent. per cent.*; what do I clear *per cent.* by ſelling them for 9 guineas *per cwt.*? *Anſ.* 50 *per cent.*

9. Laid out in a lot of muſlin 500*l.* but upon examination, 3 parts in 9 proved to be damaged, ſo that I could make but 5*s.* *per* yard of the ſame, and by ſo doing find I loſt 50*l.*: at what rate *per* ell muſt I ſell the undamaged part, ſo that I may clear 50*l.* by the whole? *Anſ.* 11*s.* 7 $\frac{2}{7}$*d.*

10. A young hare ſtarts 40 yards before a greyhound, and is not perceived by him till ſhe has been up 40 ſeconds; ſhe ſcuds away at the rate of 10 miles an hour, and the dog, on view, makes after her at the rate of 18: how long will the courſe hold, and what ground will be run over, beginning with the out-ſetting of the dog? *Anſ.* 60 $\frac{5}{22}$ *ſec. and* 530 *yards run.*

11. A traveller leaves Exeter at 8 o'clock on Monday morning, and walks towards London, at the rate of 3 miles an hour, without intermiſſion; another traveller ſets out from London at 4 o'clock the ſame evening, and walks for Exeter at the rate of 4 miles an hour conſtantly; now, ſuppoſing the diſtance between the two cities to be 130 miles, whereabouts on the road will they meet?

Anſ. 69 $\frac{3}{7}$ *miles from Exeter.*

12. A reſervoir for water has two cocks to ſupply it; by the firſt alone it may be filled in 40 minutes, and by the ſecond in 50 minutes; it hath likewiſe a diſcharging cock, by which it may, when full, be emptied in

25

25 minutes. Now, if theſe three cocks are all left open when the water comes in, in what time would the ciſtern be filled, ſuppoſing the influx and efflux of the water to be always alike? *Anſ.* 3 *h* 20 *min.*

13. A man being aſked how many ſheep he had in his drove, ſaid if I had as many more, half as many more, and ſeven ſheep and a half I ſhould have 20: how many had he? *Anſ.* 5

14. A perſon left 40 *s.* to 4 poor widows A, B, C and D; to A he left $\frac{1}{3}$, to B $\frac{1}{4}$, to C $\frac{1}{5}$ and to D $\frac{1}{6}$, deſiring the whole might be diſtributed accordingly: what is the proper ſhare of each? *Anſ. A's ſhare* 14*s.* 0 $\frac{16}{38}$*d.* *B's* 10*s.* 6 $\frac{12}{38}$*d.* *C's* 8*s.* 5 $\frac{\ }{38}$*d.* *D's* 7*s.* 0 $\frac{8}{38}$*d.*

15. How many oaken planks will floor a barn 60 $\frac{1}{2}$ feet long, and 33 $\frac{1}{2}$ wide; when the planks are 15 feet long and 15 inches wide? *Anſ.* 108

16. The amount of a ſum of money which had been put out to intereſt is 100*l.* and the principal is juſt 7 times as much as the intereſt; what is the principal? *Anſ.* 87*l.* 10*s.*

17. What number is that of which 9 is $\frac{2}{3}$ of it? *Anſ.* 13 $\frac{1}{2}$

18. A perſon dying worth 5460*l.* left his wife with child, to whom he had bequeathed, if ſhe had a ſon, $\frac{2}{3}$ of his eſtate, and the reſt to his ſon; but if ſhe had a daughter, $\frac{1}{3}$ to the daughter and the reſt to her mother: Now it happened that ſhe had both a ſon and a daughter; how muſt the eſtate be divided to anſwer the father's intention? *Anſ. The daughter's part is* 780*l.* *the ſon's* 3120*l.* *and the mother's* 1560*l.*

19. A general diſpoſing of his army into a ſquare battle, finds he has 284 ſoldiers over and above; but increaſing each ſide with one ſoldier, he wants 25 to fill up the ſquare: how many ſoldiers had he? *Anſ.* 24000

20. I would put 60 hogſheads of London beer into 30 wine pipes, and deſire to know what the caſk muſt hold.

hold that receives the difference; 231 ſolid inches being the gallon of wine, and 282 that of beer?

Anſ. 143 *gall.* 2 *qu.* 33 *rem.*

21. A tradeſman increaſed his eſtate annually $\frac{1}{3}$ part, abating 100*l.* which he uſually ſpent in his family; and at the end of 3 $\frac{1}{4}$ years, found that his net eſtate amounted to 3179*l.* 11*s.* 8*d.* what had he at his outſetting? *Anſ.* 1421*l.* 7*s.* 6$\frac{1}{2}$*d.*

22. A perſon after ſpending $\frac{2}{3}$ of his yearly income plus 10*l.* had then remaining $\frac{1}{2}$ plus 15*l.*: what was his income? *Anſ.* 150*l.*

23. There is a prize of 212*l.* 14*s.* 7*d.* to be divided amongſt a captain, 4 men, and a boy; the captain is to have a ſhare and a half; the men each a ſhare, and the boy $\frac{1}{3}$ of a ſhare: what ought each perſon to have? *Anſ. The captain* 54*l.* 14*s.* 0$\frac{2}{7}$*d. each man* 36*l.* 9*s.* 4$\frac{2}{7}$*d. and the boy* 12*l.* 3*s.* 1$\frac{3}{4}$*d.*

24. A ciſtern containing 60 gallons of water has 3 unequal cocks for diſcharging it; the greateſt cock will empty it in 1 hour; the ſecond in 2 hours, and the third in 3: in what time will it be empty if they all run together? *Anſ.* 32 $\frac{8}{11}$ *minutes*

25. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plumbs and 50 of them cheries: how many trees are there in all? *Anſ.* 600.

26. A perſon who was poſſeſſed of a $\frac{3}{5}$ ſhare of a copper-mine, ſold $\frac{3}{4}$ of his intereſt therein for 1710*l.*: what was the reputed value of the whole at the ſame rate?

Anſ. 3800*l.*

27. Suppoſe the ſea allowance for the common men to be 5 *lb's* of beef, and 3 *lb's* of biſcuit a day, for a meſs of 4 people; and that the price of the firſt is 2$\frac{1}{2}$*d. per lb.* and of the ſecond 1$\frac{1}{2}$*d.*; now, if the ſhip's company be ſuch that the meat they eat coſt the government 12 guineas *per* day; what muſt they pay for their bread *per* week? *Anſ.* 35*l.* 5*s.* 7 $\frac{1}{4}$ *d.*

28. If

28. If the ſcavenger's rate, at 1 $\frac{1}{2}$ *d.* in the pound comes to 6 *s.* 7 $\frac{1}{2}$ *d.* where they uſually aſſeſs $\frac{4}{5}$ of the rent: what will the king's tax for that houſe be at 4*s.* in the pound, rated at the full rent? *Anſ.* 13*l.* 5*s.*

29. A can do a piece of work alone in 10 days, and B in 13; ſet them both about it together, in what time will it be finiſhed? *Anſ.* 5 $\frac{15}{23}$ *days*

30. B and C together can build a boat in 18 days; with the aſſiſtance of A they can do it in 11 days; in what time would A do it by himſelf? *Anſ.* 28 $\frac{2}{7}$ *days*

31. If A can do a piece of work alone in 10 days, and A and B together in 7 days; in what time can B do it alone? *Anſ.* 23 $\frac{1}{3}$ *days*

32. A, B and C can complete a piece of work together in 12 days; C can do it alone in 24 days, and A in 34 days; in what time could B do it by himſelf? *Anſ.* 81 $\frac{3}{5}$ *days*

33. A can do a piece of work in 3 weeks; B can do thrice as much in 8 weeks, and C 5 times as much in 12 weeks: in what time can they finiſh it jointly? *Anſ.* 5 *days*, 4 *hours*

34. If a cardinal can pray a ſoul out of purgatory, by himſelf, in an hour, a biſhop in 3 hours, a prieſt in five, and a friar in 7; in what time can they pray out 3 ſouls, all praying together? *Anſ.* 1 *ho.* 47 *m.* 23$\frac{2}{11}$ *ſec.*

35. A tradeſman begins the world with 1000*l.* and finds that he can gain 1000 in five years by land trade alone; and 1000*l.* in 8 years by ſea trade alone; and likewiſe that he ſpends 1000*l.* in 2$\frac{1}{2}$ years by gaming: how long will his eſtate laſt if he follows all three? *Anſ.* 13 $\frac{1}{3}$ *years*

36. Bought 120 oranges at 2 a penny, and 120 more at 3 a penny, and ſold them all together at 5 for 2*d.*: what did I gain or loſe by the bargain? *Anſ. Loſt* 4*d.*

37. A water tub holds 147 gallons; the pipe uſually brings in 14 gallons in 9 minutes; the tap diſcharges, at a medium, 40 gallons in 31 minutes; now, ſuppoſing theſe

these both to be carelessly left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5, finding the water running, shuts the tap, and is solicitous to know in what time the tap will be filled after this accident, in case the water continues to flow from the main. *Ans. The tub will be full at 3 min.* 48 $\frac{21}{31}$ *sec. after* 6.

38. Part 1500*l.*; give B 72*l.* more than A, and C 112*l.* more than B. *Ans. A's share is* 414 $\frac{2}{3}$ *l. B's* 486 $\frac{2}{3}$ *l. C's* 598 $\frac{2}{3}$ *l.*

39. A and B venturing equal sums of money clear by joint trade 154*l.*; by agreement A was to have 8 *per cent.* because he spent his time in the execution of the project; and B was only to have 5: what was A allowed for his trouble? *Ans.* 35*l.* 10*s.* 9 $\frac{3}{13}$ *d.*

40. A, B and C are to share 100000*l.* in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively; but C's part being lost by his death, it is required to divide the whole sum properly between the other two.

Ans. A's part is 57142 $\frac{282}{329}$, *and B's* 42857 $\frac{47}{329}$

41. A stationer sold quills at 11*s.* a thousand, by which he cleared $\frac{3}{8}$ of the money; but growing scarce he raised them to 13*s.* 6*d.* a thousand; what did he clear *per cent.* by the latter price? *Ans.* 96*l.* 7*s.* 3$\frac{3}{11}$*d.*

42. Required the least number that can be divided by 1, 2, 3, 4, 5, 6, 7, 8 and 9 without leaving a remainder. *Ans.* 2520

43. Suppose a man has a calf, which at the end of three years begins to breed, and afterwards brings a female calf every year; and that each calf begins to breed in like manner at the end of three years, bringing forth a cow calf every year; and that these last breed in the same manner, &c.; to determine the owners whole stock at the end of 20 years. *Ans.* 1278

ARITH-

ARITHMETICAL TABLES.

NUMERATION Table.

Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
9	8	7	6	5	4	3	2	1
	9	8	7	6	5	4	3	2
		9	8	7	6	5	4	3
			9	8	7	6	5	4
				9	8	7	6	5
					9	8	7	6
						9	8	7
							9	8
								9

PENCE Table.

d		s	d
20	is	1	8
30		2	6
40		3	4
50		4	2
60		5	-
70		5	10
80		6	8
90		7	6
100		8	4
110		9	2
120		10	-

d		s
12	is	1
24		2
36		3
48		4
60		5
72		6
84		7
96		8
108		9
120		10

4 farthings ... make ... 1 penny.
12 pence .... 1 ſhilling
20 ſhillings .... 1 pound.

A TABLE of the Cuſtomary Weight of Goods

	lb
A Firkin of Butter .... is	56
A Firkin of Soap	64
A Barrel of Pot Aſhes	200
A Barrel of Anchovies	30
A Barrel of Candles	120
A Barrel of Soap	256
A Barrel of Butter	224
A Fother of Lead is 19½ cwt.	
A Stone of Iron	14
A Stone of Butchers Meat	8
A Gallon of Train Oil	7½
A Faggot of Steel	120
A Stone of Glaſs	5
A Seam of Glaſs is 24 Stone or	120

L denotes pounds, s ſhillings, and d pence.
¼ is 1 farthing, or 1 quarter of any thing.
½ ... a half-penny, or a half of any thing.
¾ ... 3 farthings, or 3 quarters of any thing.

TROY Weight.

24 Grains make 1 penny wt.
20 penny wts. .. 1 ounce
12 ounces ..... 1 pound

By this weight are weighed jewels, gold, ſilver, corn, bread and liquors.

APOTHECARIES Weit.

20 grains make 1 ſcruple
3 ſcruples ..... 1 dram
8 drams .... 1 ounce
12 ounces ...... 1 pound

Apothecaries uſe this weit. in compounding their medi=cines, but they buy their drugs by Avoirdupoiſe weight.

BREAD.

	lb	oz	dr
A Peck Loaf weighs	17	6	1
A Half Peck	8	11	-
A Quartern	4	5	8

HAY.

56 lb's old hay } make a truſs
60 lb's new hay }
36 truſses .... a load

20

1
20

20
24

80
40

10 | 480
48

Avoirdupoise Weight.

16 drams	make	1 ounce
16 ounces		1 pound
28 pounds		1 quarter
4 quarters		1 hund.d weight
20 hund.d w.t		1 ton .

By this weight are weighed all things of a coarse or drossy nature: such as butter, cheese, flesh, grocery wares, & all mettals, except gold and silver.

Coal Measure.

4 pecks	make	1 bushel
9 bushels		1 Vat or Strike
36 bushels		1 chaldron
21 chaldrons		1 score .

Dry Measure.

2 pints	make	1 quart
2 quarts		1 pottle
2 pottles		1 gallon
2 gallons		1 peck
4 pecks		1 bushel
8 bushels		1 quarter
5 quarters		1 wey or load
4 bushels		1 coomb
5 pecks		1 bush. water mea.r
10 coombs		1 wey
2 weys		1 last .

By this measure salt, lead, ore, oysters, corn & other dry goods are measured.

Cubic Measure.

1728 cubic inches		1 cubic foot
27 cubic feet		1 cubic yard .

Ale and Beer Measure.

2 pints	make	1 quart
4 quarts		1 gallon
8 gallons		1 firkin of Ale
9 gallons		1 firkin of Beer
2 firkins		1 kilderkin
2 kilderkins		1 barrel
3 kilderkins		1 hogshead
3 barrels		1 butt .

Note: the ale gallon contains 282 cubic inches. In London the ale firkin contains 8 gallons, & the Beer firkin 9, other measures being in the same proportion.

Long Measure.

3 barley-corns	make	1 inch
12 inches		1 foot
3 feet		1 yard
6 feet		1 fathom
$5\frac{1}{2}$ yards		1 pole
40 poles		1 furlong
8 furlongs		1 mile
3 miles		1 league
60 geographical miles, or		
$69\frac{1}{2}$ statute miles		1 degree
360 degrees		circumference of the earth .

Wool Weight.

7 lb's	make	1 clove
2 cloves		1 stone
2 stone		1 tod
$6\frac{1}{2}$ tods		1 wey
2 weys		1 sack
12 sacks		1 last .

WINE Meaſure.

2 pints	make	1 quart
4 quarts		1 gallon
42 gallons		1 tierce
63 gallons		1 hogſhead
84 gallons		1 puncheon
2 hogſheads		1 pipe or butt
2 pipes		1 tun

By this meaſure, brandies, ſpirits, perry, cyder, mead, vinegar, oil and honey are meaſured.

Note. 231 ſolid inches make a gallon, and 10 gallons make an anchor.

CLOTH Meaſure.

4 nails	make	1 quarter
4 quarters		1 yard
3 quarters		1 ell flemiſh
5 quarters		1 ell engliſh
6 quarters		1 ell french

SQUARE Meaſure.

144 ſquare inches	1 ſquare foot
9 ſquare feet	1 ſquare yard
30¼ ſquare yards	1 ſquare pole
40 ſquare poles	1 ſquare rood
4 ſquare roods	1 ſquare acre

TIME.

60 ſeconds	make	1 minute
60 minutes		1 hour
24 hours		1 day
7 days		1 week
4 weeks		1 month

13 months, 1 day, and 6 hours, or 365 days, and 6 hours, 1 year nearly.

The number of days in each month.

Thirty days hath September,
April, June, and November:
February hath twenty eight alone,
And all the reſt have thirty one;
But leap year, coming once in four,
Doth give to February one day more.

PRACTICE Table.

The aliquot parts of a pound.

s.	d.		
10	-	is	half
6	8		third
5	-		fourth
4	-		fifth
3	4		ſixth
2	6		eighth
2	-		tenth
1	8		twelfth

The aliquot parts of a ſhilling.

d		
6	is	half
4		third
3		fourth
2		ſixth
1½		eighth
1		twelfth

Multiplication Table.

2 times	2	is	4
	3		6
	4		8
	5		10
	6		12
	7		14
	8		16
	9		18
	10		20
	11		22
	12		24
3 times	3	is	9
	4		12
	5		15
	6		18
	7		21
	8		24
	9		27
	10		30
	11		33
	12		36
4 times	4	is	16
	5		20
	6		24
	7		28
	8		32
	9		36
	10		40
	11		44
	12		48
5 times	5	is	25
	6		30
	7		35
	8		40
	9		45
	10		50
	11		55
	12		60
6 times	6	is	36
	7		42
	8		48
	9		54
	10		60
	11		66
	12		72
7 times	7	is	49
	8		56
	9		63
	10		70
	11		77
	12		84
8 times	8	is	64
	9		72
	10		80
	11		88
	12		96
9 times	9	is	81
	10		90
	11		99
	12		108
10 times	10	is	100
	11		110
	12		120
11 times	11	is	121
	12		132
12 times	12	is	144

FINIS.